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Algebraic Complexity Theory

SS 2014, Exercise Sheet #5

EXERCISE 9:

a) Formulate and prove an extension/generalization of Theorem 3.1 (Baur-Strassen) in the script to the structure $\mathcal{S} = \big(\mathbb{R}, C, (+, -, \times, \div, \exp, \log, \sqrt{\ })\big)$.

b) Let $\mathbb{F}$ denote a field of characteristic 0. Suppose $\det_n :\subseteq \mathbb{F}^{n\times n} \to \mathbb{F}$ can be calculated (on a Zariski dense subset) by some straight-line program P_n over $\mathcal{S} = \big(\mathbb{F}, C, (+, -, \times, \div)\big)$. Conclude that $\mathrm{Inv}_n : \mathrm{GL}(\mathbb{F}, n) \to \mathrm{GL}(\mathbb{F}, n), A \mapsto A^{-1}$ can be calculated (on a Zariski dense subset) by a straight-line program over $\mathcal{S}$ of length $\mathcal{O}(|P_n|)$.
Hint: Differentiate Laplace's expansion of the determinant.

EXERCISE 10:
Prove the first two claims from Lemma 3.5 in the script:

a) $p \in \mathbb{F}[[X]]$ has a multiplicative inverse in $\mathbb{F}[[X]]$ iff $p(0) \neq 0$;
in which case $q := 1/p$ is given by $q_0 = 1/p_0$ and inductively $q_n = -\sum_{m=1}^{n} p_m \cdot q_{n-m}/p_0$.

b) Suppose $\tilde{q} \in \mathbb{F}[[X]]$ satisfies $p \cdot \tilde{q} \equiv 1 \pmod{X^n}$. Then $\tilde{\tilde{q}} := \tilde{q} \cdot (2 - p \cdot \tilde{q}) \cdot p \equiv 1 \pmod{X^{2n}}$; cmp. Exercise 8c).