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Algebraic Complexity Theory

SS 2014, Exercise Sheet #4

EXERCISE 7:

a) Prove the following weakening of Fact 2.3g) from the script:
There exists an infinite subset of $\mathbb{R}$ algebraically independent over $\mathbb{Q}$.

b) The *Thue–Siegel–Roth Theorem* asserts that any irrational algebraic number α can*not* be approximated too well in the following sense:
For every $\varepsilon > 0$, only finitely many coprime integers p, q satisfy $\left|\alpha - p/q\right| < 1/q^{2+\varepsilon}$.
Conclude that $\sum_n 2^{-3^n}$ is transcendental.

EXERCISE 8:

a) Verify that the N–dimensional discrete Fourier transform $\mathcal{F}_N$ satisfies

$$\mathcal{F}_N\big(\vec{x} * \vec{y}\big) \;=\; \big(\mathcal{F}_N(\vec{x})\big) \cdot \big(\mathcal{F}_N(\vec{y})\big), \quad \text{where} \quad \vec{x} * \vec{y} \;:=\; \Big(\sum_{\ell=0}^{N-1} x_\ell \cdot y_{k-\ell \bmod N}\Big)_{0 \le k < N}$$

denotes (discrete circular) convolution and “$\cdot$” componentwise multiplication.

b) Describe a straight-line program over $\mathcal{S}_1 = \big(\mathbb{C}, \mathbb{C}, (+, \times_\lambda : |\lambda| \le 1)\big)$ of length $\mathcal{O}(N \cdot \log N)$ computing the *inverse* of the N–dimensional discrete Fourier transform, where $N = 2^n$.

c) Recall Newton’s Method for finding a root of the (non-linear) function f by iterating $x \mapsto x - \frac{f(x)}{f'(x)}$ and devise an algorithm for approximating the reciprocal $1/x$ to given $x \neq 0$.