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Solutions due: 2014-04-22*

# Algebraic Complexity Theory

## SS 2014, Exercise Sheet #1

**EXERCISE 1:**
Investigate the power $d$ of asymptotic growth $t(n) \in \Theta(n^d)$ for $t : \mathbb{N} \to \mathbb{R}$ satisfying the following recursion: $t(n) = a \cdot t(\lceil n/b \rceil) + c \cdot n$ for $1 \leq b < a \leq c$.

**EXERCISE 2:**
Let $\ell(n)$ denote the number of additions sufficient to produce the number $n \in \mathbb{N}$, starting with 1. We prove $\ell(n) \leq \log_2(n) + \mathcal{O}\big(\frac{\log_2 n}{\log_2 \log_2 n}\big)$ as follows:

a) Fix $\lambda \approx \log_2 \log_2 n$ to be later chosen exactly.
Then all integers $1, \ldots, 2^\lambda - 1$ together can be calculated within a total of $2^\lambda$ additions.

b) Calculating $a_0 + a_1 \cdot 2^\lambda + a_2 \cdot 2^{2\lambda} + \cdots + a_d 2^{d \cdot \lambda}$ from $a_0, \ldots, a_d$ suffices with $(\lambda + 1) \cdot d$ additions. Hint: Horner

c) Now choose $d := \lceil \log_2 n / \lambda \rceil$ and $\lambda :\approx \log_2 \log_2 n - 2 \log_2 \log_2 \log_2 n$.

d) Describe an algorithm asserting $\ell(2^{16} - 1) \leq 19$.

**EXERCISE 3:**
Devise an algorithm computing the complex polynomial

$$\sqrt{2} \cdot X^6 + i \cdot X^5 + X^4 + (1 - \sqrt{2}) \cdot X^2 - i \cdot X - 2$$

from $X$ and some complex constants using only 5 multiplications and 6 additions.

*Please consider the following poll: `http://doodle.com/bbrmh86zqdpe6kus`