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Complexity Theory

WS 2011/2012, Exercise Sheet #8

EXERCISE 21:

a) Adapt Exercise 5 to DTMs with separate *read-only* input tape.

b) Adapt Exercise 12c) to show $\mathcal{NL} \subseteq \mathcal{P}$.

So far we have considered polynomial-time reductions "$\preccurlyeq_p$". Now define "$A \preccurlyeq_L B$" as the existence of a total function f such that $x \in A \Leftrightarrow f(x) \in B$ computable in logarithmic space where neither input nor output tape account for the space consumption.

c) Let $\mathcal{FL}$ denote the class of functions computable within logarithmic space. Prove: $\mathcal{FL} \subseteq \mathcal{FP}$.

d) Prove that $\mathcal{FL}$ is closed under composition.

e) Conclude: i) $A \preccurlyeq_L B \Rightarrow A \preccurlyeq_p B$, ii) $A \preccurlyeq_L B \in \mathcal{L} \Rightarrow A \in \mathcal{L}$,
iii) $A \preccurlyeq_L B \in \mathcal{NL} \Rightarrow A \in \mathcal{NL}$ iv) $A \preccurlyeq_L B \wedge B \preccurlyeq_L C \Rightarrow A \preccurlyeq_L C$.

EXERCISE 22:

a) Prove: $\textsc{Cyclic} := \{\langle G\rangle : \text{directed graph } G \text{ contains a cycle}\} \in \mathcal{NL}$.

b) Prove: $2\textsc{Sat} \in \text{co}\mathcal{NL}$. Hint: Analyze Exercise 9c+d).

c) Prove that $2\textsc{Sat}$ is co$\mathcal{NL}$-complete.

A graph $G = (V,E)$ is called *bipartite* if its vertices can be partitioned into two classes with no edges running within one class; formally:

$$\exists V_1,V_2 : V = V_1 \cup V_2 \wedge V_1 \cap V_2 = \emptyset \quad \forall (u,v) \in E : (u \in V_1 \wedge v \in V_2) \vee (v \in V_1 \wedge u \in V_2) \ .$$

d) Prove that a graph is bipartite iff it contains no cycle of odd length.

e) Prove: $\textsc{Bipartite} := \{\langle G\rangle : \text{graph } G \text{ is bipartite}\} \in \text{co}\mathcal{NL}$.

f) Prove that $\textsc{Bipartite}$ is co$\mathcal{NL}$-complete.

g) Can you prove (directly) that $2\textsc{Sat}, \textsc{Bipartite} \in \mathcal{NL}$?

EXERCISE 23:

$$\textsc{dirPath} := \big\{\langle G,s,t\rangle : \text{directed graph } G = (V,E) \text{ contains a path from } s \in V \text{ to } t \in V\big\}$$

is called the *directed* path problem; similarly $\textsc{undirPath}$ for the *undirected* counterpart. OMER REINGOLD has received the 2006 ACM Grace Murray Hopper Prize for his 2004 proof that undirected path can be decided in *deterministic* logarithmic space: $\textsc{undirPath} \in \mathcal{L}$.

a) Prove that every $L \in \mathcal{P}$ satisfies $L \preccurlyeq_p \textsc{undirPath}$.

b) Does this answer the important open question "$\mathcal{L}$ *versus* $\mathcal{P}$"? Why/why not?
Specify two $\mathcal{L}$-complete problems.