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Complexity Theory

WS 2011/2012, Exercise Sheet #7

EXERCISE 18:
A function $s : \mathbb{N} \to \mathbb{N}$ is called *constructible* if a Turing machine can compute it unarily (i.e. the mapping $1^n \mapsto 1^{s(n)}$) within time $\mathcal{O}\big(s(n)\big)$.

a) Prove that every $s(n) \geq n$ expressible as arithmetic expression over $1, +, \times, x \mapsto 2^x$ is constructible.

b) What changes if we replace 'unary' with 'binary', i.e. require $\mathrm{bin}(n) \mapsto 1^{s(n)}$ or $1^n \mapsto \mathrm{bin}\big(s(n)\big)$ or $\mathrm{bin}(n) \mapsto \mathrm{bin}\big(s(n)\big)$ to be computable in time $\mathcal{O}\big(s(n)\big)$?

c) Where and how exactly did the hypothesis for $n \mapsto s(n)$ to be constructible enter in the proof of Savitch's Therorem?

d) Show that this prequisite can be weakened to the function $1^n \mapsto \mathrm{bin}\big(s(n)\big)$ being computable within space $\mathcal{O}\big(s(n)^{\mathbf{2}}\big)$.

e) What running time does the deterministic simulation in the proof of Savitch's Theorem take?

EXERCISE 19:
Let $s : \mathbb{N} \to \mathbb{N}$ be constructible. Prove:

a) $\mathsf{NSPACE}\big(s(n)\big) \subseteq \mathrm{co}\mathsf{NSPACE}\big(s(n)^2\big)$

b) $\mathrm{co}\mathsf{NSPACE}\big(s(n)\big) \subseteq \mathsf{NSPACE}\big(s(n)^2\big)$

c) Conclude $\mathsf{NPSPACE} = \mathrm{co}\mathsf{NPSPACE}$.

d) Can you modify this argument to show $\mathcal{NP} = \mathrm{co}\mathcal{NP}$?

EXERCISE 20:
Consider the language

$$\textsc{dirPath} = \left\{ \langle G, s, t \rangle \;\middle|\; \begin{array}{l} G = (V, E) \text{ is a directed graph, } s, t \in V \text{ and} \\ \text{there exists a directed path from } s \text{ to } t \text{ in } G. \end{array} \right\}.$$

Describe and analyze an algorithm deciding DIRPATH while using, in addition to the space occupied by the input, only $\mathcal{O}\big(\log(n)^2\big)$ additional tape cells. What is its running time?