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Complexity Theory

WS 2011/2012, Exercise Sheet #1

EXERCISE 3:
Consider the DTM $\mathcal{M} = (Q, \Sigma, \Gamma, \delta)$ where $Q = \{s, q_1, q_2, q_3, q_+, q_-\}$, $\Sigma = \{0, 1\}$, $\Gamma = \Sigma \cup \{\triangleright, \sqcup\}$ and transition function $\delta : (Q \setminus \{q_-, q_+\}) \times \Gamma \to Q \times \Gamma \times \{\mathsf{L}, \mathsf{N}, \mathsf{R}\}$ as defined as follows:

$$\begin{array}{llll}
(s,\triangleright) \mapsto (s,\triangleright,\mathsf{R}), & (q_2,\triangleright) \mapsto (q_2,\triangleright,\mathsf{R}), & (s,0) \mapsto (q_1,\sqcup,\mathsf{R}), & (q_2,0) \mapsto (q_-,0,\mathsf{N}), \\
(s,1) \mapsto (q_-,1,\mathsf{N}), & (q_2,1) \mapsto (q_3,\sqcup,\mathsf{L}), & (s,\sqcup) \mapsto (q_+,\sqcup,\mathsf{N}), & (q_2,\sqcup) \mapsto (q_2,\sqcup,\mathsf{L}), \\
(q_1,\triangleright) \mapsto (q_1,\triangleright,\mathsf{R}), & (q_3,\triangleright) \mapsto (q_3,\triangleright,\mathsf{R}), & (q_1,0) \mapsto (q_1,0,\mathsf{R}), & (q_3,0) \mapsto (q_3,0,\mathsf{L}), \\
(q_1,1) \mapsto (q_1,1,\mathsf{R}), & (q_3,1) \mapsto (q_3,1,\mathsf{L}), & (q_1,\sqcup) \mapsto (q_2,\sqcup,\mathsf{L}), & (q_3,\sqcup) \mapsto (s,\sqcup,\mathsf{R}).
\end{array}$$

Determine the sequence of direct successor configurations on $\mathcal{M}$ on inputs `010101` and `0011`. Which language does $\mathcal{M}$ accept/decide?

EXERCISE 4:

a) Formalize the DTM for `PALIN` from the lecture by specifying its transition function δ.

b) Analyze its asymptotic running time.

c) This DTM contains a bug: identify and correct the error!

d) Using the 'programming techniques' from the lecture, describe semi-informally a DTM counting in binary the length of its input, i.e. computing the following function:

$$(x_1, \ldots, x_n) \mapsto \operatorname{bin}(n) \in \{0,1\}^*, \quad \text{where} \quad \operatorname{bin}\left(\sum_{j=0}^{m-1} y_j 2^j + 2^m\right) := (y_0, y_1, \ldots, y_{m-1}, 1)$$

Analyze its asymptotic running time and the length of the output.

e) Describe semi-informally a DTM deciding $\{0^n 1^n : n \in \mathbb{N}\} \subseteq \{0,1\}^*$ in time $\mathcal{O}(n \cdot \log n)$.

EXERCISE 5:

a) Let $\mathcal{M} = (Q, \Sigma, \Gamma, \delta)$ be a DTM and $\vec{w} \in \Sigma^*$. Prove: If $\mathcal{M}$ visits, on input $\vec{w}$, at most s tape cells while making more than $s \cdot |Q| \cdot |\Gamma|^s$ steps, then $\mathcal{M}$ runs indefinitely.

b) Let $\mathcal{M}$ be a DTM accepting $L \subseteq \Sigma^*$ in space $\mathcal{O}\big(s(n)\big)$ where $s : \mathbb{N} \to \mathbb{N}$ is computable in time $2^{\mathcal{O}(s(n))}$. Describe a DTM $\mathcal{N}$ deciding L in time $T_{\mathcal{N}}(\vec{w}) \leq 2^{\mathcal{O}(s(n))}$.