restart;

Aufgabe 1: Ausgleichsrechnung

with(*plots*) :
with(*Statistics*) :
$X := \langle 90, 180, 270, 360, 450, 540, 630, 720 \rangle;$

$$\begin{bmatrix} 90 \\ 180 \\ 270 \\ 360 \\ 450 \\ 540 \\ 630 \\ 720 \end{bmatrix} \tag{1.1}$$

$Y := \langle 299.72, 723.33, 1178.98, 1711.08, 2161.69, 2260.98, 2418.65, 2502.74 \rangle;$

$$\begin{bmatrix} 299.72 \\ 723.33 \\ 1178.98 \\ 1711.08 \\ 2161.69 \\ 2260.98 \\ 2418.65 \\ 2502.74 \end{bmatrix} \tag{1.2}$$

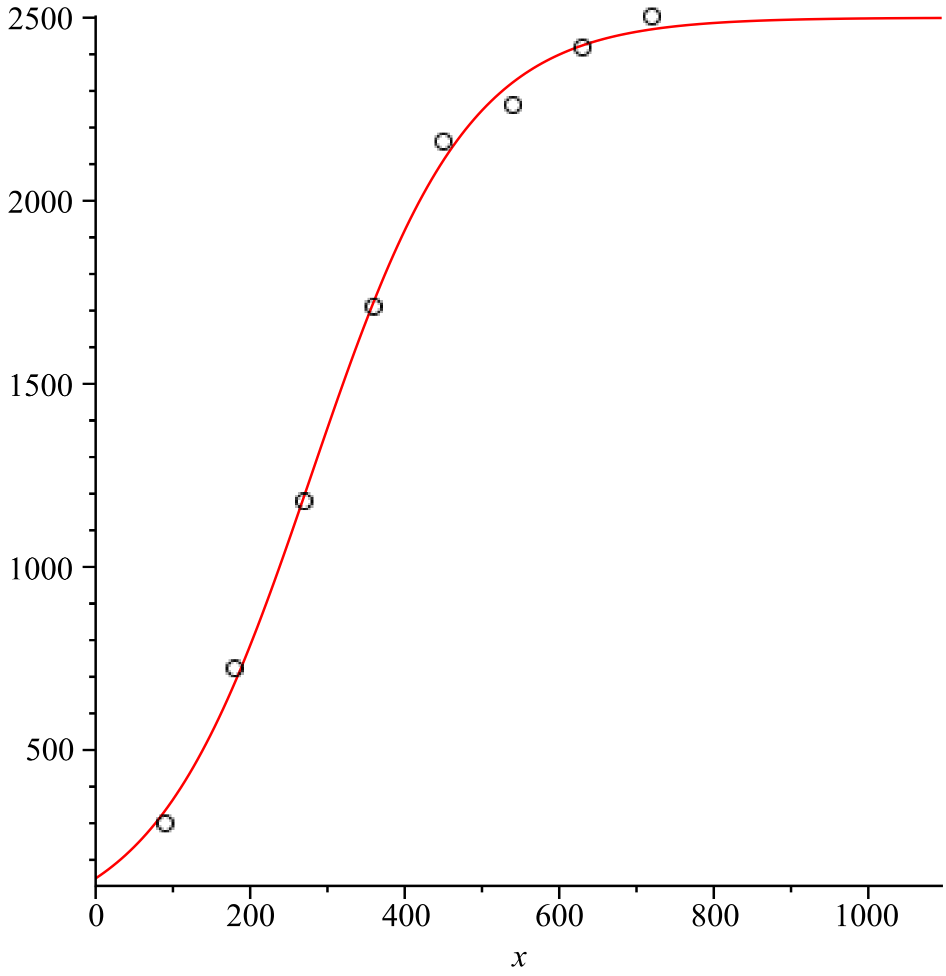
$f := \frac{2500 \cdot \mathrm{e}^{\frac{a}{100} \cdot x}}{b + \mathrm{e}^{\frac{a}{100} \cdot x}};$

$$\frac{2500\, \mathrm{e}^{\frac{1}{100} a x}}{b + \mathrm{e}^{\frac{1}{100} a x}} \tag{1.3}$$

$fit := Fit(f, X, Y, x);$

$$\frac{2500\, \mathrm{e}^{0.00987599839115128\, x}}{15.7280539249178 + \mathrm{e}^{0.00987599839115128\, x}} \tag{1.4}$$

display(*plot*(*fit*, $x = 0..3 \cdot 365$), *pointplot*(*X*, *Y*, *symbol* = *circle*, *symbolsize* = 15));



Aufgabe 2: Gleichungssysteme

$solve([x^2 + y^2 = 16, x + y = p], [x, y], Explicit);$

$$\left[\left[x = \frac{1}{2}\,p - \frac{1}{2}\,\sqrt{-p^2 + 32}, y = \frac{1}{2}\,p + \frac{1}{2}\,\sqrt{-p^2 + 32}\right], \left[x = \frac{1}{2}\,p + \frac{1}{2}\,\sqrt{-p^2 + 32}, y = \frac{1}{2}\,p - \frac{1}{2}\,\sqrt{-p^2 + 32}\right]\right] \quad (2.1)$$

Aufgabe 3: Prozeduren

```
numbers := proc(n)
 local i;
 for i to n do
  print(i);
 end do; # Anmerkung: end do ≙ od
end proc;
```

$$\textbf{proc}(n)\ \textbf{local}\ i;\ \textbf{for}\ i\ \textbf{to}\ n\ \textbf{do}\ print(i)\ \textbf{end do}\ \textbf{end proc} \tag{3.1}$$

$numbers(4);$

$$1$$
$$2$$
$$3$$
$$4 \tag{3.2}$$

Aufgabe 4: Sequenzen

a)

$a := 3, 4, 5;$

$$3, 4, 5 \tag{4.1.1}$$

$b := NULL, 1, 9;$

$$1, 9 \tag{4.1.2}$$

$c := a, b;$

$$3, 4, 5, 1, 9 \tag{4.1.3}$$

$c := c, 42;$

$$3, 4, 5, 1, 9, 42 \tag{4.1.4}$$

b)

```
myfacs :=proc(n)
  local L, i;
  L := NULL;
  for i while i! < n do
    L := L, i!;
  end do; # Anmerkung: end do ≙ od
  return L;
end proc;
```

```
proc(n)                                                              (4.2.1)
    local L, i;
    L := NULL; for i while factorial(i) < n do L := L, factorial(i) end do; return L
end proc
```

$myfacs(7);$

$$1, 2, 6 \tag{4.2.2}$$

Aufgabe 5: Polynome

$g := x \to -x^4;$

$$x \rightarrow -x^4 \quad (5.1)$$

$h := x \rightarrow -x^4 - x^3 + 10 \cdot x^2 + 3;$

$$x \rightarrow -x^4 - x^3 + 10\, x^2 + 3 \quad (5.2)$$

a)

if $\sqrt{4} > 0$ **then** 1 **else** 0 **end**;

$$1 \quad (5.1.1)$$

if $\sqrt{3} > 0$ **then** 1 **else** 0 **end**;

```
Error, cannot determine if this expression is true or false: 
0 < 3^(1/2)
```

if $is(\sqrt{4} > 0)$ **then** 1 **else** 0 **end**;

$$1 \quad (5.1.2)$$

if $is(\sqrt{3} > 0)$ **then** 1 **else** 0 **end**;

$$1 \quad (5.1.3)$$

b)

```
maxima := proc(f)
            local L, z, el;
            L := NULL;
            z := [fsolve(f'(x) = 0, x)];
            for el in z do
               if f''(el) < 0 then L := L, el;
               end if;
            end do;
            return [L];
 end proc;
```

(5.2.1)

```
proc(f)
   local L, z, el;
   L := NULL;
   z := [fsolve(diff(f(x), x) = 0, x)];
   for el in z do if eval(diff(f(x), x, x), x = el) < 0 then L := L, el end if end do;
   return [L]
end proc
```

maxima(h);

$$[-2.642294643, 1.892294643] \quad (5.2.2)$$

c)

```
maxima := proc(f)
```

```
        local L, z, el;
        L := NULL;
        if degree( f(x) ) > 5 then
          z := [ fsolve( f'(x) = 0, x ) ];
        else
          z := [solve( f'(x) = 0, x, DropMultiplicity ) ];
        end if;
        for el in z do
            if is( f''(el) < 0 ) then L := L, el;
            end if;
        end do;
        return [L];
end proc;
```

```
proc( f )                                                                    (5.3.1)
    local L, z, el;
    L := NULL;
    if 5 < degree( f(x) ) then
        z := [ fsolve( diff( f(x), x ) = 0, x ) ]
    else
        z := [solve( diff( f(x), x ) = 0, x, DropMultiplicity ) ]
    end if;
    for el in z do if is( eval( diff( f(x), x, x ), x = el ) < 0 ) then L := L, el end if end do;
    return [L]
end proc
```

maxima(h);

$$\left[-\frac{3}{8} + \frac{1}{8}\sqrt{329}, -\frac{3}{8} - \frac{1}{8}\sqrt{329} \right] \quad \textbf{(5.3.2)}$$

d)

```
maxima := proc( f )
        local L, z, el, n, ii;
        L := NULL;
        n := degree( f(x) );
        if n ≤ 5 then
          z := [solve( f'(x) = 0, x, DropMultiplicity ) ];
          for el in z do
            for ii from 2 to n do
              if D^(ii)( f )( el ) ≠ 0 then
                if ii mod 2 = 0 and is( D^(ii)( f )( el ) < 0 ) then
                  L := L, el;
                end if;
                break;
              end if;
```

```
                end do;
            end do;
        else
            z := [fsolve(f'(x) = 0, x)];
            for el in z do
                if is(f''(el) < 0) then L := L, el;
                end if;
            end do;
        end if;
        return [L];
end proc;
```

(5.4.1)

```
proc(f)
    local L, z, el, n, ii;
    L := NULL;
    n := degree(f(x));
    if n <= 5 then
        z := [solve(diff(f(x), x) = 0, x, DropMultiplicity)];
        for el in z do
            for ii from 2 to n do
                if `@@`(D, ii)(f)(el) <> 0 then
                    if mod(ii, 2) = 0 and
                    is(`@@`(D, ii)(f)(el) < 0) then
                        L := L, el
                    end if;
                    break
                end if
            end do
        end do
    else
        z := [fsolve(diff(f(x), x) = 0, x)];
        for el in z do
            if is(eval(diff(f(x), x, x), x = el) < 0) then L := L, el end if
        end do
    end if;
    return [L]
end proc
```

e)

```
maxima(g);
```

$$[0] \tag{5.5.1}$$

```
maxima(h);
```

$$\left[-\frac{3}{8}+\frac{1}{8}\sqrt{329}, -\frac{3}{8}-\frac{1}{8}\sqrt{329}\right] \quad \textbf{(5.5.2)}$$

f)

Nullstellen sind im Allgemeinen nur für Polynome von Grad ≤ 4 analytisch bestimmbar. Die Ableitung eines Polynoms von Grad 5 ist ein Polynom von Grad 4.