Introduction to Mathematical Software Exercise 7

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Problem 1 Integration ☆☆★

Find an antiderivative of $f(x) = \tan(x) \cdot \sin(x)$.

Problem 2 Solving Equations Numerically ☆☆★

Find an approximate solution of $x^2 = \sin(x)$ in the interval $\left[\frac{1}{2}, 1\right]$.

Problem 3 Procedures: Digit Sum ☆★★

Write a procedure that returns the digit sum of a given natural number n.

Problem 4 Procedures: Perfect Numbers ☆★★

A natural number is called *perfect* if it is equal to the sum of its proper divisors, e.g. $6 = 1 + 2 + 3$. Write a procedure that returns the first n perfect numbers as a list. Test your procedure for all $n \in \{1, 2, 3, 4\}$.

Problem 5 Mandelbrot Set ★★★

The Mandelbrot set is the set of complex numbers $c \in \mathbb{C}$ for which the sequence $z_0, z_1, z_2, \ldots$ with $z_0 = 0$ and $z_{n+1} = z_n^2 + c$ is bounded.
Hint: To solve this exercise, it might be helpful to have a look at the solution of the image processing exercise.

a) Use the `Create`-command from the `ImageTools`-package to create an image img with height 201, width 301 and background color white.

b) Write a function t that maps the pixel (x, y) to the complex number $\left(\frac{1}{100} \cdot x - \frac{201}{100}\right) + \left(\frac{-1}{100} \cdot y + \frac{101}{100}\right) \cdot I$. (This is done for scaling purposes.) Verify that $t(1,1) = -2 + I$, $t(301, 201) = 1 - I$ and $t(201, 101) = 0$.

c) Write a procedure m that checks if the series $z_0, z_1, z_2, \ldots$ is bounded for a given complex number c. Initialize z_0 with 0.0 to disable slow exact arithmetic. Do 50 iterations. Return 1 if the absolute value of an element of the series is greater than 50, otherwise return 0.

d) Colorize all pixels (x, y) in the following way: $img[y, x] := m(t(x, y))$.

(*Remark:* 1 means white, 0 means black. For images, the first index is y.)

e) Have a look at the image using the `View`-command.