Introduction to Mathematical Software Exercise 3

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Problem 1 Curve Fitting

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In the first exercise, you explored the growth of a Goron. The biologist has done some measurements and wants to improve the parameters. The model function is:

$$f(x) = \frac{2500 \cdot e^{\frac{a}{100} \cdot x}}{b + e^{\frac{a}{100} \cdot x}}$$

The results of the measurements are:

x	90	180	270	360	450	540	630	720
f(x)	299,72	723,33	1178,98	1711,08	2161,69	2260,98	2418,65	2502,74

Find appropriate parameters a and b. Make a plot.
Hint: Have a look at the notes of the first lecture.

Problem 2 Systems of Equations

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Solve the following (non-linear) system of equations for x and y:

$$x^2 + y^2 = 16$$
$$x + y = p$$

Check help topic `solve,details` to find out how the parameter `Explicit` can help you to display the solutions in a convenient way.

Problem 3 Procedures

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Write a procedure that successively prints all natural numbers from 1 to n to screen.

Problem 4 Sequences

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a) Check what sequences do in Maple. Find out what the following inputs do.

1) `a:=3,4,5;`
2) `b:=NULL,1,9;`
3) `c:=a,b;`
4) `c:=c,42;`

Hint: `NULL` is the empty sequence.

b) Write a Maple procedure that lists all factorials that are less than a given natural number.

Example: input: 7, output: [1,2,6].

Hint: Remember that loops may have the following form: `for` ... `while` ... `do` ... `end do`.

Problem 5 Polynomials ☆★★

For this exercise you may assume that the method *maxima* shall only be applied to polynomials.

a) As a preliminary consideration compare the results of the following inputs:
   i) **if** $\sqrt{4} > 0$ **then** 1 **else** 0 **end**;
   ii) **if** $\sqrt{3} > 0$ **then** 1 **else** 0 **end**;
   iii) **if** $is\left(\sqrt{4} > 0\right)$ **then** 1 **else** 0 **end**;
   iv) **if** $is\left(\sqrt{3} > 0\right)$ **then** 1 **else** 0 **end**;

b) Modify the procedure *maxima* so that it returns a list of all maximum points.

c) Modify the procedure so that it returns exact maximal positions for polynomials of degree ≤ 5. Return numerical maximal positions for polynomials of higher degree.

d) For certain polynomials (e.g. $f(x) = x^4$) the method from c) does not work correctly. Fix this problem at least for polynomials of degree ≤ 5 by checking derivatives of higher order.

e) Test your method with the following functions:
   i) $g(x) = -x^4$
   ii) $h(x) = -x^4 - x^3 + 10 \cdot x^2 + 3$

f) Can you imagine why we only do exact calculations for polynomials of degree ≤ 5?

Problem 6 An Application: Image Processing (Part 2) ★★★

f) Open the Maple document that you saved last week. Press the "!!!"-button in order to make Maple aware of all your former inputs.

g) Rotate your monochrome picture by 90° counterclockwise using the `Rotate`-command. Take a look at the result.

h) Create a new image with the same dimensions as the rotated image. Write a (nested) loop that colors your image in a chessboard pattern. *Hint:* Helpful commands: `Create`, `Height`, `Width`.

i) We want to reduce the noise in our image using the so-called *median-filter*. Create a new image with the same dimensions as the rotated image. Do not modify the pixels of the original image! In the new Image, we set all inner pixels to the *median* of the original pixel and the 8 pixels around it. (This may take some seconds.) You can calculate the median by applying the `Statistics[Median]`-command to a list of these 9 pixels. Of course this does not work for the border-pixels of the image. For this exercise, it is perfectly ok to ignore them completely. Again, take a look at the result to see whether the filter worked as intended.

j) Save the file. This exercise will be continued next week.