Introduction to Mathematical Software Exercise 2

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Problem 1 Curve Sketching ☆☆★

Let

$$f(x) = \frac{x^2 + 5 \cdot x - 12}{2 \cdot x^2 - 12 \cdot x + 16} \qquad \text{and} \qquad g(x) = \pi^2 \cdot \left(e^{f(x) \cdot e^{-2}} - 1 \right).$$

a) Define $f(x)$ und $g(x)$. Verify that $g(12) \approx 1,74$.

b) Plot $g(x)$ for $x \in [-10, 10]$. Use the same interval as range of your plot. Have a look at help topic `plot,options` in order to find out how to do that. Also try to find out how to avoid ugly discontinuities in your plot.

c) Find the domain of g.

d) Find the zeros of g.

e) Find local minima and maxima of g.

f) Calculate the area under the graph of g (numerically) between the first two zeros (there should be at least two zeros).

g) Determine both left-hand and right-hand limit of g at all discontinuities. Also determine the limits for $x \to \infty$ and $x \to -\infty$.

Problem 2 `plot` and `plot3d` ☆☆★

a) Let

$$f(x) = \sin(\pi \cdot x^3), \qquad g(x) = e^{-x^2}.$$

Plot $f(x)$ and $g(x)$ with $x \in [-2, 2]$. Try to plot f and g into one picture.

b) Plot $h(x) = x^2$ and its derivative from $x = 0$ to ∞. Enjoy the result!

c) How can you plot the function MonkeySaddle$(x, y) = x^3 - 3 \cdot x \cdot y^2$, $x \in [-2, 2]$, $y \in [-2, 2]$? Take a look into Maple's help in order to find out more about plotting.

Problem 3 Solving Systems of Linear Equations ☆☆★

Try to solve the following systems of linear equations. What is happening?

$$\begin{aligned} 2\cdot x+8\cdot y+4\cdot z&=7\\ 6\cdot x+2\cdot y+4\cdot z&=9\\ x+z&=8 \end{aligned}$$

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$$\begin{aligned} 2\cdot x+8\cdot y+4\cdot z&=7\\ 6\cdot x+2\cdot y+4\cdot z&=9\\ 3\cdot x+8\cdot y+5\cdot z&=9\\ x+z&=8 \end{aligned}$$

Hint: You do not need any matrices here. Just have a look at the `solve`-command.

Problem 4 Lists and Sets in Maple ☆★★

a) Explain the difference between lists and sets in Maple.

b) Use Maple to find the common divisors of 23545800, 25491186 and 229420674. *Hint:* How can the package `numtheory` help you?

c) Solve the equation

$$x^4-4\cdot x^3\cdot\pi+\frac{26}{9}\cdot x^2\cdot\pi^2+\frac{4}{9}\cdot x\cdot\pi^3-\frac{1}{3}\cdot\pi^4=0$$

and then let Maple evaluate the function `sin` for all solutions by using

i) `map`,

ii) the element-wise operator $\sim$.

Hint: Remember using lists.

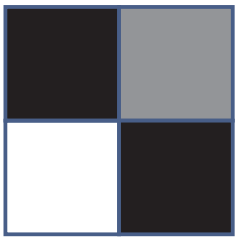
Problem 5 Procedures ☆★★

a) Copy the procedure *maxima* from the lecture notes.

b) Apply it to the following examples:

1. $f(x)=-x^2$
2. $g(x)=-x^4$
3. $h(x)=-x^4-x^3+10\cdot x^2+x+3$
4. $i(x)=\sin(x)$

Did you expect the result? Try to find an explanation if this is not the case.



$$\begin{pmatrix} 0 & 0,5 \\ 1 & 0 \end{pmatrix}$$

Figure 1: Image with 4 pixels and corresponding matrix

Problem 6 An Application: Image Processing (Part 1) ☆☆☆

Important: In order to solve this exercise, you will need some commands from the package `ImageTools`. You might notice that many of its commands also appear in other packages (e.g. `AudioTools`). Do not get confused when consulting the help.
Hint: For debugging-purposes, the commands `View`, `Preview` and `Write` might be useful. Check out what they do!

a) Open a new document in *Document Mode*. Load the package `ImageTools`. You will need some of its commands for this exercise.

b) Go to the course website. Download `Image1.jpg` from the "Exercises"-section. Remember the full path where you save it (e.g. `/home/username/Image1.jpg`).

c) Load the Image into a variable using the `Read`-command.
(*Hint for Windows-users:* Use this notation for paths: `C:/path/to/my/file/Image1.jpg`)

d) Convert the image to grayscale using the `ToGrayscale`-command. Now you have a 2D-array of floating point numbers between 0 and 1. Each element of the array represents one pixel. 0 means black, 1 means white (cf. figure 1). If your array is `A`, you can access the pixels by `A[row,column]`, e.g. `A[10,10]`.

e) Have a look at the result. Save the file. This exercise will be continued next week.