Linear Algebra II
Tutorial Sheet no. 13

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Exercise T1 (Warm-up: Determinant revisited)
We consider the real vector space V of symmetric, 2×2 real matrices.

(a) Prove that $\det : V \to \mathbb{R}$ is a a quadratic form.

(b) Determine the matrix of the associated bilinear form with respect to the basis

$$\mathscr{B} = \left(B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad B_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right).$$

(c) Determine the principal axes and sketch the sets

$$\{\mathbf{v} \in V \mid \det \mathbf{v} = 1\}, \quad \{\mathbf{v} \in V \mid \det \mathbf{v} = -1\}.$$

(as subsets of $\mathbb{R}^3$, when every matrix is identified with its coordinates w.r.t. the basis $\mathscr{B}$).

Exercise T2 (A quadric up to rotation/translation)
Consider the quadratic $\mathbb{X}$ given by $3x^2 + 3y^2 - 2xy + 20x - 12y + 40 = 0$. Our goal is to find the principal axes and describe the graph of $\mathbb{X}$.

(a) Regarding the quadratic part of the above equation as a quadratic form, diagonalise the associated symmetric bilinear form to obtain a basis for which the cross term xy vanishes.

(b) Working in this new basis, eliminate the linear terms by a translation.

(c) Describe $\mathbb{X}$.

Exercise T3 (Slicing a quadric)
Consider the quadric $\mathbb{X}_{\lambda,\mu}$ in $\mathbb{R}^3$ defined by

$$\mathbb{X}_{\lambda,\mu} := \{\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3 : \lambda(x_1^2 + x_2^2) + \mu x_3^2 = 1\},$$

where λ and μ are real parameters.

(a) Determine the intersection of every $\mathbb{X}_{\lambda,\mu}$ with the plane defined by $x_3 = c \in \mathbb{R}$.

(b) Prove that $\mathbb{X}_{\lambda,\mu}$ can be obtained by rotating the set

$$\mathbb{X}'_{\lambda,\mu} := \{\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 0,\ \lambda x_2^2 + \mu x_3^2 = 1\}$$

about the x_3-axis.

(c) For each pair of values

	λ	μ
1.	-1	1
2.	1	-1
3.	2	1

,

sketch $\mathbb{X}_{\lambda,\mu}$ and $\mathbb{X}'_{\lambda,\mu}$.