Linear Algebra II
Tutorial Sheet no. 11

TECHNISCHE UNIVERSITÄT DARMSTADT

Prof. Dr. Otto
Dr. Le Roux
Dr. Linshaw

Summer term 2011
June 20, 2011

Exercise T1 (Warmup: Skew-hermitian and skew-symmetric matrices)

A matrix $A \in \mathbb{C}^{(n,n)}$ is called skew-hermitian if $A^{+} = -A$. Similarly, in the real case, $A \in \mathbb{R}^{(n,n)}$ is called skew-symmetric if $A = -A^{t}$.

(a) Show that any skew-hermitian or skew-symmetric matrix is normal.

(b) Conclude that for any skew-hermitian matrix A, there exists a unitary matrix U such that $UAU^{-1} = D$, where D is diagonal.

(c) Let $A \in \mathbb{C}^{(n,n)}$ be skew-hermitian. What can you say about the eigenvalues of A?

Exercise T2 (Self-adjoint and normal endomorphisms)

Let V be a finite dimensional euclidean or unitary space and φ an endomorphism of V. Prove the following.

(a) If V is euclidean, then

$$\varphi \text{ is self-adjoint} \iff V \text{ has an orthonormal basis consisting of eigenvectors of } \varphi.$$

(b) If V is unitary, which one of the implications from (a) does not hold?

(c) If V is unitary, then

$$\varphi \text{ is normal} \iff V \text{ has an orthonormal basis consisting of eigenvectors of } \varphi.$$

Exercise T3 (Orthogonal diagonalisability)

Find an *orthogonal* matrix C such that the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

is transformed into a diagonal matrix by $C^{-1}AC = C^{t}AC$. Which property of A guarantees that you can find such a C?

[Hint: The charactaristic polynomial is $p_A = (X-1)^2(X-4)$]