Linear Algebra II
Tutorial Sheet no. 2

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Exercise T1 (Geometric characterisation of linear maps by eigenvalues)

Give a geometric description of all the endomorphisms of $\mathbb{R}^3$ with the following sets of eigenvalues:

(a) $\lambda_1 = -1, \lambda_2 = 0, \lambda_3 = 1$

(b) $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$

(c) $\lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 2$

Note that you cannot assume anything about the corresponding eigenvectors other than that they form a basis (why?).

Exercise T2 (Eigenvalues and eigenvectors over $\mathbb{R}$ and $\mathbb{C}$)

Let A be the 3×3-matrix $\begin{pmatrix} 0 & -1 & 4 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) Determine the characteristic polynomial of the matrix A.

(b) Find all real eigenvalues of A and the corresponding eigenvectors of the map $\varphi : \mathbb{R}^3 \to \mathbb{R}^3$ with $\varphi(x) = Ax$.

(c) Find all eigenvalues for the corresponding map $\varphi : \mathbb{C}^3 \to \mathbb{C}^3$ with $\varphi(x) = Ax$ and give a basis of each eigenspace.

Exercise T3 (Diagonalisation)

Consider the matrix $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$ over $\mathbb{R}$.

(a) Determine all eigenvalues of A and corresponding eigenvectors.

(b) Find a regular matrix C such that $D = C^{-1}AC$ is a diagonal matrix.

(c) Calculate A^6.

(d) Find a “positive square root” of A, i.e., find a matrix R with non-negative eigenvalues such that $R^2 = A$

(e) Check that $t \mapsto e^{tA}\mathbf{v}_0$ solves the differential equation $\frac{d}{dt}\mathbf{v}(t) = A\mathbf{v}(t)$ with initial value $\mathbf{v}(0) = \mathbf{v}_0$.

Exercise T4 (Eigenvalues of nilpotent maps)

Let V be a vector space of dimension greater than 0, and let $\varphi : V \to V$ be a nilpotent endomorphism, that is, an endomorphism such that $\varphi^k = \mathbf{0}$ for some $k \in \mathbb{N}$.

(a) Show that 0 is the only possible eigenvalue of φ.

(b) Show that 0 is an eigenvalue of φ.