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Advanced Complexity Theorie

SS 2011, Exercise Sheet #11

EXERCISE 17:

a) Describe a branching tree on n inputs over $\mathcal{S} = \big(\{0,1\}, (), (), (<)\big)$ sorting within depth $\mathcal{O}(n)$.

b) Let $[n] := \{1, \ldots, n\}$ and suppose that branching tree T on n inputs over $\mathcal{S} = \big([n], (), (), (<)\big)$ sorts. Show that T then also sorts over any other totally ordered structure $\mathcal{S}' = \big(S, (), (), (<)\big)$.

c) Let T_n denote a family branching tree on n inputs over $\mathcal{S} = \big([n], (), (), (<)\big)$. Suppose that T_n sorts *approximately* in the sense that the function $\vec{f} : [n]^n \to [n]^n$ it computes satisfies

$$f_1(\bar{x}) \leq f_2(\bar{x}) \leq \ldots \leq f_n(\bar{x}) \text{ and } \forall y \in S: \ \text{Card}\{j : x_j = y\} \ = \ \text{Card}\{j : f_j(x_1, \ldots, x_n) = y\}$$

(not necessarily for every, but only) for at least an 2^{-n}-fraction of all permutations $(x_1, \ldots, x_n)$. Conclude that, still, T_n must have asymptotic depth at least $\Omega(n \cdot \log n)$.

EXERCISE 18:
Recall Definition 6.3 from the script.

a) Describe a branching tree over $\mathcal{S} = \big(\mathbb{R}, \mathbb{R}, (+, \times), (<)\big)$ for the membership problem induced by $\mathcal{H}$. Estimate its depth and size in terms of d and $n = |\mathcal{H}|$.

b) Express the Knapsack problem as (the integer restriction of) a membership problem for affine hyperplanes.

c) Show that the arrangement from a) contains a cell with exponentially many facets.

d) Repeat b)+c) for the Travelling Salesperson problem.