Martin Ziegler

Issued on 2011-07-1
To be submitted on 2011-07-7 by noon in S2/15-206

Advanced Complexity Theorie

SS 2011, Exercise Sheet #10

EXERCISE 15:

a) Formulate and prove an extension/generalization of Theorem 5.8 (Baur-Strassen) in the script to the structure $\mathcal{S} = \big(\mathbb{R}, C, (+,-,\times,\div,\exp,\log,\sqrt{\ })\big)$.

b) Let F denote a field of characteristic 0. Suppose $\det_n : F^{n\times n} \to F$ can be calculated by some straight-line program P_n over $\mathcal{S} = \big(F, C, (+,-,\times,\div)\big)$. Conclude that $\mathrm{Inv}_n : \mathrm{GL}(F,n) \to \mathrm{GL}(F,n)$, $A \mapsto A^{-1}$ can be calculated by a straight-line program over $\mathcal{S}$ of length $\mathcal{O}(|P_n|)$.

EXERCISE 16:

a) Describe an algorithm for approximating the inverse of a given matrix or floating point number a using only additions and multiplications by finding a root x of $f(x) = a - x^{-1}$. Compare it with Theorem 5.9a+b) from the script.

b) A Toeplitz Matrix of size $n \times n$ has the form

$$T = \begin{pmatrix} a_0 & a_{-1} & a_{-2} & \cdots & a_{-n+2} & a_{-n+1} \\ a_1 & a_0 & a_{-1} & \ddots & \ddots & a_{-n+2} \\ a_2 & a_1 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & a_{-1} & a_{-2} \\ \vdots & & \ddots & a_1 & a_0 & a_{-1} \\ a_{n-1} & \cdots & \cdots & a_2 & a_1 & a_0 \end{pmatrix}$$

with $2n-1$ parameters $a_{-n+1}, a_{-n+2}, \ldots, a_{-1}, a_0, a_1, \ldots, a_{n-2}, a_{n-1}$. Show that, given complex numbers for these parameters and for $x_1, \ldots, x_n$, one can calculate $T \cdot \vec{x}$ within $\mathcal{O}(n \cdot \log n)$ arithmetic steps.
Hint: First suppose $a_{-n+j} = a_j$ for $1 \le j < n$ and recall Exercise 13.

c) Show that a given complex $n \times n$ Vandermode matrix can be multiplied to a given vector in $\mathcal{O}(n \cdot \log^2 n)$ arithmetic steps.