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Advanced Complexity Theorie

SS 2011, Exercise Sheet #8

EXERCISE 13:
Recall Exercise 10 on the n-dimensional discrete Fourier-transform $\mathcal{F}_n$ and let $\mathcal{S} = \big(\mathbb{C}, \mathbb{S}^1, (+,\times)\big)$.

a) For $\vec{x} = (x_0, \ldots, x_{n-1}), \vec{y} \in \mathbb{C}^n$, their (circular) convolution is defined as

$$\vec{x} \otimes \vec{y} \;:=\; \big(\textstyle\sum_{\ell=0}^{n-1} x_k \cdot y_{k-\ell \bmod n}\big)_{k=0\ldots n-1} \;\in\; \mathbb{C}^n \quad .$$

Show that $\mathcal{F}_n(\vec{x} \otimes \vec{y}) = (\mathcal{F}_n\vec{x}) \cdot (\mathcal{F}_n\vec{y})$ componentwise.

b) Describe a straight-line program over $\mathcal{S}$ of length $\mathcal{O}(n \cdot \log n)$ solving the following problem: Given the lists $p_0, \ldots, p_{n-1}, q_0, \ldots, q_{n-1}$ of coefficients of two polynomials $p, q \in \mathbb{C}[X]$ of $\deg(p), \deg(q) < n$, calculate the list of coefficients of their product $p \cdot q$.

c) Describe a straight-line program over $\mathcal{S}$ calculating the product of m univariate polynomials of degree $< n$. What length do you achieve?

d) Describe a straight-line program over $\mathcal{S}$ calculating the sum of m univariate rational functions of degree (of both numerator and denominator) $< n$. What length do you achieve?