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Advanced Complexity Theorie

SS 2011, Exercise Sheet #6

EXERCISE 9:

a) Show that Proposition 5.3b) in the script is optimal.

b) Compare the straight-line complexity of 2^n-1 over $\big(\mathbb{N},(1),(+)\big)$ with that over $\big(\mathbb{N},(1),(+,-)\big)$.

c) Prove that any straight-line program computing $N \in \mathbb{N}$ over $\big(\mathbb{N},(1),(+,-,\times)\big)$ has length at least $\log_2 \log_2 N$.

d) Is Claim c) asymptotically optimal? Prove or improve!

e) Let $\mathcal{S} = \big(S,(c_i)_{i\in I},(f_j)_{j\in J}\big)$ be a structure of finite signature. Bound the number of straight-line programs over $\mathcal{S}$ of length N in n variables asymptotically as $N \to \infty$.

f) Can *every* $N \in \mathbb{N}$ be computed by a straight-line program over $\big(\mathbb{N},(1),(+,-,\times)\big)$ of length $\mathcal{O}(\log_2 \log_2 N)$? Prove or disprove!