Martin Ziegler

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Advanced Complexity Theorie

SS 2011, Exercise Sheet #4

EXERCISE 6:

a) Show that the following languages are in $\mathsf{DSPACE}(n)$:
i) SAT, ii) $\mathsf{Hamilton}$, iii) Clique with edges listed as adjacencies

b) Fix $k \in \mathbb{N}$ and let $L \in \mathsf{DSPACE}\big(f(n)^k\big)$ with $n \in \mathcal{O}\big(f(n)\big)$ and $L' := \big\{\langle \vec{x}, 0^{|\vec{x}|^k}\rangle : \vec{x} \in L\big\}$.
Then $L \preccurlyeq_{\mathrm{p}} L' \preccurlyeq_{\mathrm{p}} L$ and $L' \in \mathsf{DSPACE}\big(f(n)\big)$.

c) Prove $\mathcal{NP} \neq \mathsf{DSPACE}(n)$.
(Hint: Hierarchy Theorem plus closure under polynomial-time reductions)

EXERCISE 7:
Totality is the question of whether a given program terminates on *every* input:

$$T \quad := \quad \big\{\langle \mathcal{M}\rangle : \forall \overline{x} \in \Sigma^* : \langle \mathcal{M}, \overline{x}\rangle \in H\big\}$$

a) Prove that neither T nor its complement is semi-decidable

b) but its complement is semi-decidable relative to H.

c) To every L semi-decidable relative to H there exists a decidable R such that

$$L \quad = \quad \big\{\overline{x} \,|\, \exists \overline{y} \forall \overline{z} \colon\ \langle \overline{x}, \overline{y}, \overline{z}\rangle \in R\big\} \ . \tag{1}$$

Hint: Recall the polynomial hierarchy…

d) If R is decidable, then L according to c) is many-one reducible to the complement of T.

e) Conclude that T is not semi-decidable relative to H.