

Advanced Complexity Theorie

SS 2011, Exercise Sheet #3

EXERCISE 5:

Recall the proof of Time-versus-Space (Script, Lemma 2.4c).

- a) How much space does the simulating machine use if the number B of phases/blocks is chosen as $\Theta(T^{1/4})$?
- b) What for $B = \Theta(T^{1/2})$?
- c) Determine the asymptotic growth of $P_\ell(n)$ according to Exercise 4b).
- d) Let \mathcal{M} denote a 1-tape DTM making T steps on input $\vec{x} \in \Sigma^n$. As in Lemma 2.4b) from the script, divide this computation into $B \in \mathbb{N}$ phases of $\mathcal{O}(T/B)$ steps, and the tape into B blocks of length $\mathcal{O}(T/B)$ each. Define a graph on B vertices such that vertex φ receives an edge from vertex ψ iff the computation of $\mathcal{M}(\vec{x})$ in phase φ depends on the contents of blocks produced last modified in phase ψ .

Prove that this graph is planar and bound its indegree.

- e) Assert the following improvement of Time-versus-Space for 1-tape DTMs:

If $t(n) \geq n$ is constructible in space $\mathcal{O}(\sqrt{t(n)})$, then $\text{DTIME}_1(t(n)) \subseteq \text{DSpace}(\sqrt{t(n)})$.