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Advanced Complexity Theorie

SS 2011, Exercise Sheet #2

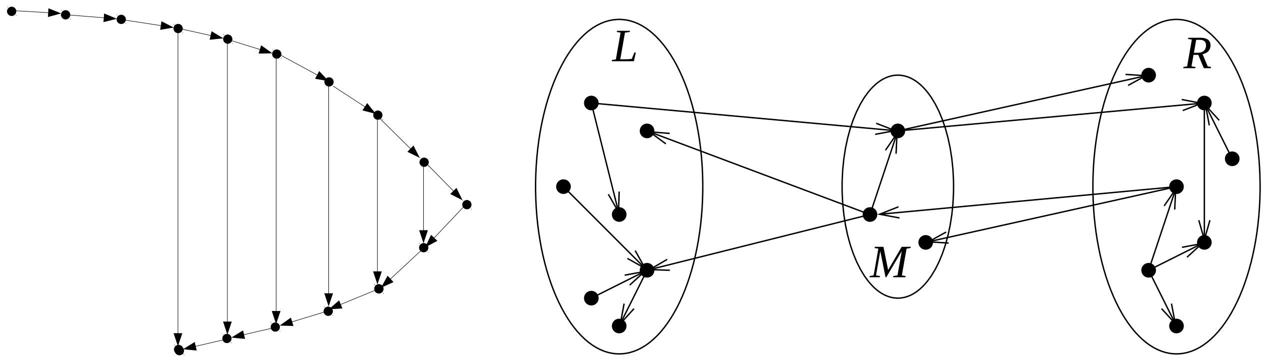
EXERCISE 3:
For $n \leq m \in \mathbb{N}$, consider the directed acyclic harp graph depicted in Figure a) below. Prove:

a) It can be played with 3 markers in $\mathcal{O}(n \cdot m)$ steps

b) It can be played with $n+1$ markers in $\mathcal{O}(n+m)$ steps

c) It can be played with $3 \leq k \leq n+1$ markers in $\mathcal{O}\left(\frac{n \cdot m}{k-2}\right)$ steps

d) It cannot be played with 3 markers in asymptotically less than $n \cdot m$ steps.



a) Harp graph for $n = 6$ and $m = 9$; b) Illustrating the planar separator theorem.

A graph (V,E) is called planar if it can be drawn in the plane with (not necessarily straight-line) edges intersecting only in the vertices they are incident to.
The Planar Separator Theorem (Lipton, Tarjan, SIAM J.Appl.Math. 1979) asserts that every planar (V,E) admits a partition $V = L \uplus M \uplus R$ such that there are no edges between L and R (see Figure b), and $|L|, |R| \leq \frac{2}{3} \cdot |V|$, and $|M| \leq \sqrt{8 \cdot |V|}$.

EXERCISE 4:

a) Argue that the complete graph on four vertices is planar
but the complete graph on fives vertices is not.

b) Let $P_\ell(n)$ denote the least number of pebbles sufficient to play *every* planar DAG on n vertices with indegree at most ℓ. Prove $P_\ell(n) \leq P(\frac{2}{3}n) + \ell + \sqrt{8n}$.