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Advanced Complexity Theorie

SS 2011, Exercise Sheet #1

EXERCISE 1:
Let $L := \{\vec{x}\,0^{|\vec{x}|}\,\vec{x} : \vec{x} \in \Sigma^*\}$.

a) Describe a 1-tape Turing machine deciding L in time $\mathcal{O}(n^2)$.

b) Describe a 2-tape Turing machine deciding L in time $\mathcal{O}(n)$.

EXERCISE 2:

a) Describe a sequence $\vec{x}_n$ of binary strings of length $|\vec{x}_n| = n$ with $K(\vec{x}_n) \leq \mathcal{O}(\log|\vec{x}_n|)$.
How about $\vec{x}$ with $K(\vec{x}) \leq \mathcal{O}(\log\log|\vec{x}|)$?

b) Prove that, asymptotically, 'almost every' binary string of length n has Kolmogorov Complexity at least $n/2$. Determine the complexity of a random binary string of length n as $n \to \infty$.

c) It is not known whether transcendental number π is normal (with respect to base 2, say).
Let $\vec{x}_n = (1,1,0,0,1,0,0,1,0,0,0,0,1,1,1,1,1,1,0,1,1,0,1,0,\ldots) \in \{0,1\}^n$ denote the sequence of the first n binary digits of π. Estimate the Kolmogorov Complexity of $\vec{x}_n$
i) in case π is normal, and ii) in case it is not.

d) Let $r \in [0,1)$ denote an arbitrary real number and $\vec{x}_n$ the sequence of the first n binary digits of r and $\vec{y}_n$ the sequence of the first n *decimal* digits of r, each separately encoded in binary.

Compare the Kolmogorov Complexity of $\vec{x}_n$ with that of $\vec{y}_n$ up to an additive constant independent of n.

Would you prefer this lecture and/or exercises to be moved to a different date?
Please indicate your preferences at `doodle.de/68qe9ygftr7av66t`