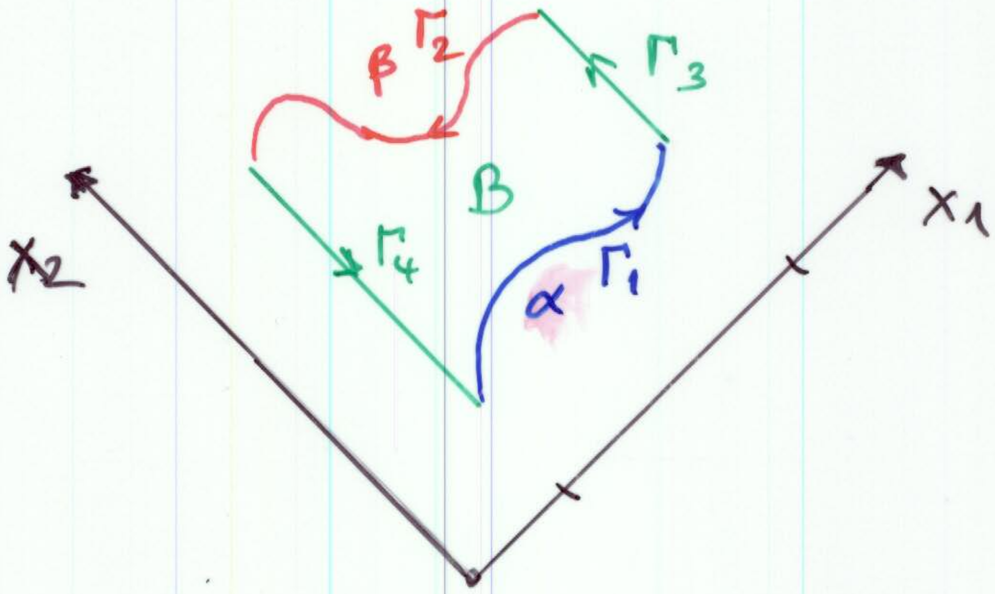


26.4 $\int_B \frac{\partial f}{\partial x_2} = -\int_\Gamma f(\vec{x})\, dx_1$ allef

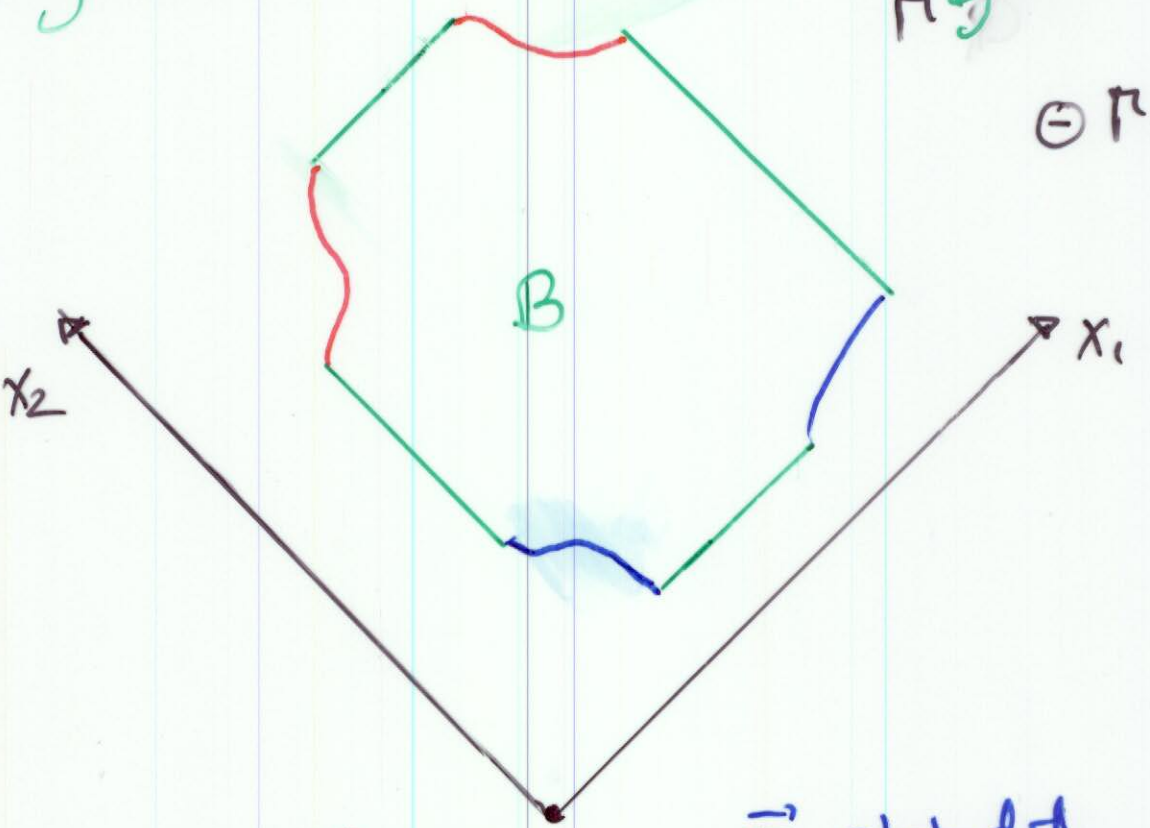
26.5 $\int_B \frac{\partial f}{\partial x_1} = \int_\Gamma f(\vec{x})\, dx_2$ allef

26.6

B grünes Bereich

$\Gamma \circlearrowleft$

$\ominus\Gamma \circlearrowright$



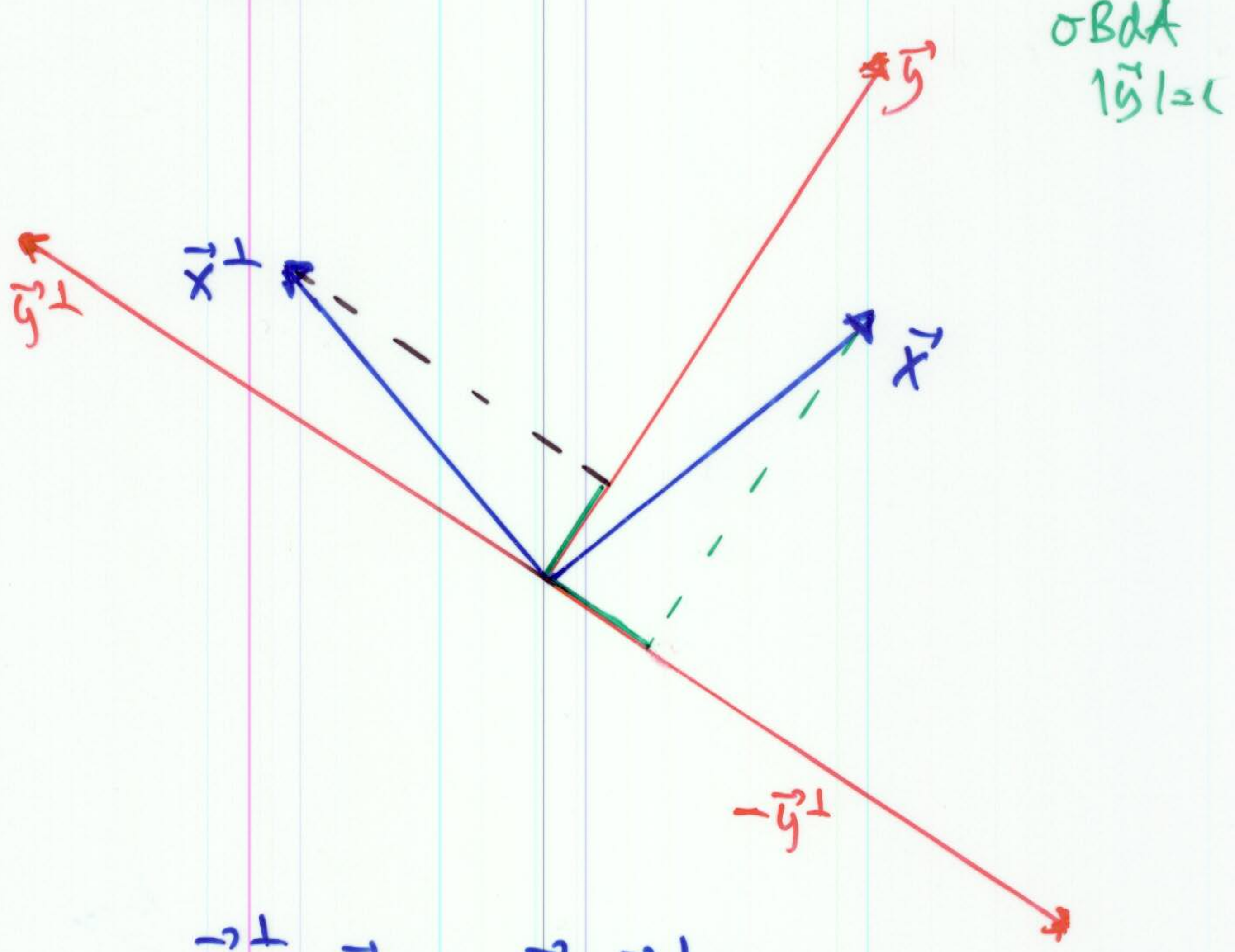
Satz 26.3 Green $\quad$ $\vec{F}$ stet. diff

$$\int_\Gamma \vec{F}\cdot d\vec{x} = \int_\Gamma F_1\,dx_1 + \underbrace{\int_\Gamma F_2\,dx_2}_{= -\int_{\ominus\Gamma} F_2\,dx_2}$$

$$= -\int_B \frac{\partial F_1}{\partial x_2} - -\int_B \frac{\partial F_2}{\partial x_1}$$

$$= \int_B \underbrace{\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2}}_{=:\,\mathrm{rot}\,\vec{F}}$$

26.7



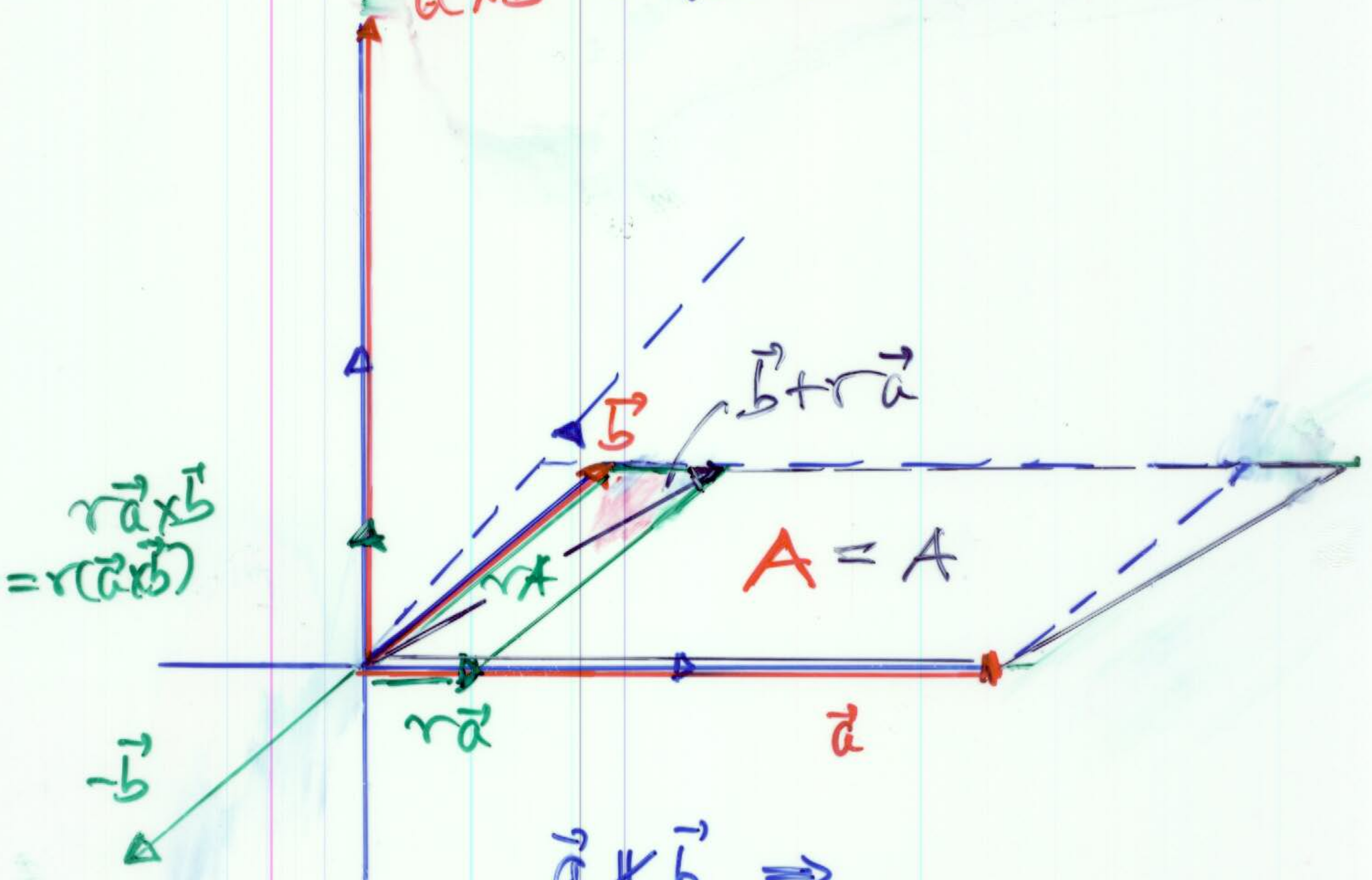
$$\vec{x}^\perp \cdot \vec{y} = -\vec{x} \cdot \vec{y}^\perp$$

$$\vec{x} \hat{=} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \vec{x}^\perp \hat{=} \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$$

$$\det \begin{pmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{pmatrix} = x_1^2 + x_2^2$$

27.1 Vektor product

$$\color{red}{\vec{a}\times\vec{b}} = \vec{a}\times(\vec{b}+r\vec{a})$$



$\vec{a} \nparallel \vec{b} \Rightarrow$

$\vec{a}\times\vec{b} \perp \vec{a}, \vec{b}$

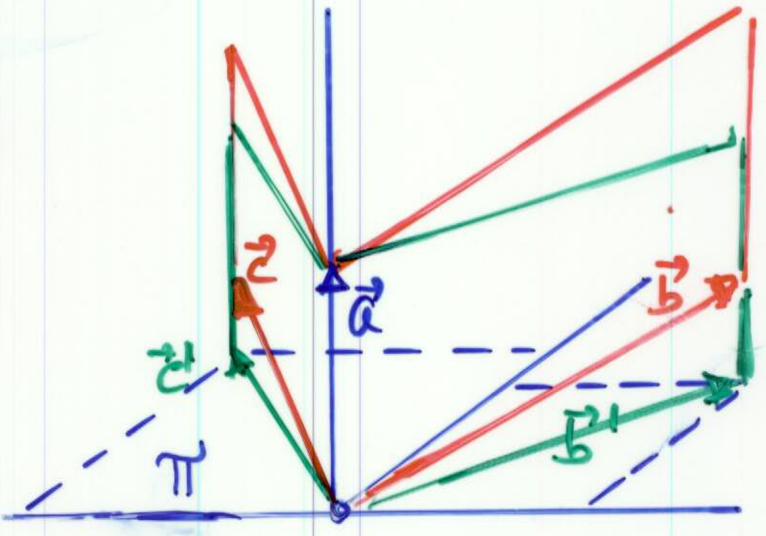
$\vec{a}, \vec{b}, \vec{a}\times\vec{b}$ pos. or.

$\|\vec{a}\times\vec{b}\| = A = \|\vec{a}\|\,\|\vec{b}\|\,|\sin\alpha|$

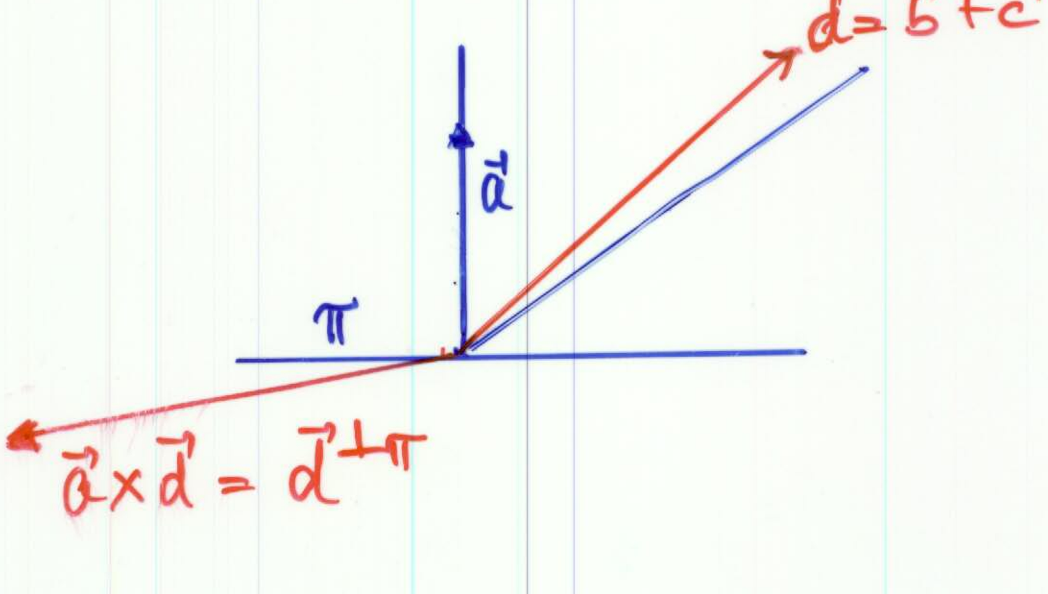
$\vec{b}\times\vec{a}$
$= -\vec{a}\times\vec{b}$

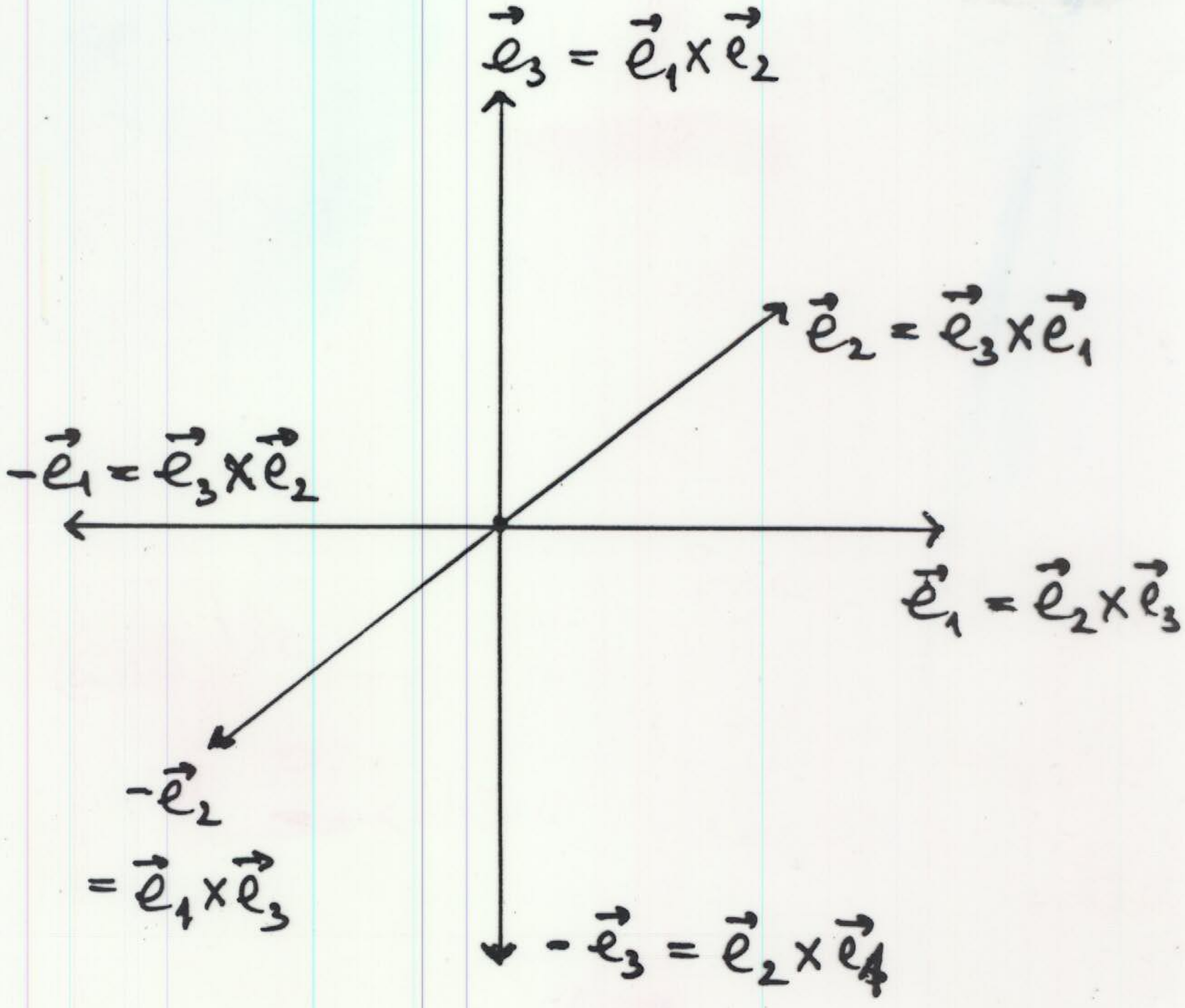
$\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a}\times\vec{b} = \vec{0}$

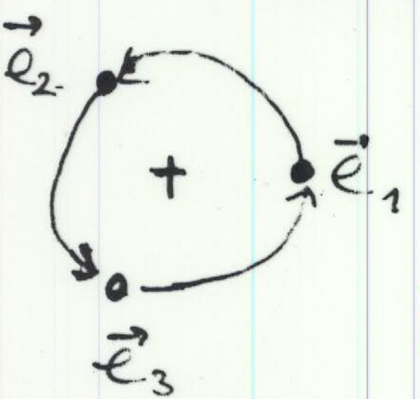
$|\vec{a}|=1$



$$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times(\vec{b}'+\vec{c}')$$
$$= (\vec{b}'+\vec{c}')^{\perp\pi}$$
$$= \vec{b}'^{\perp\pi} + \vec{c}'^{\perp\pi}$$
$$= \vec{a}\times\vec{b}' + \vec{a}\times\vec{c}'$$
$$= \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$$







Scherungsinvarianz

$$\vec{a} \times (\vec{b} + r\vec{a}) = \vec{a} \times \vec{b}$$

$$\left.\begin{aligned} r(\vec{a} \times \vec{b}) &= r\vec{a} \times \vec{b} = \vec{a} \times r\vec{b} \\ \vec{a} \times (\vec{b} + \vec{c}) &= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} \\ (\vec{b} + \vec{c}) \times \vec{a} &= \vec{b} \times \vec{a} + \vec{c} \times \vec{a} \end{aligned}\right\} \text{linear}$$

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} \quad \text{alternierend}$$

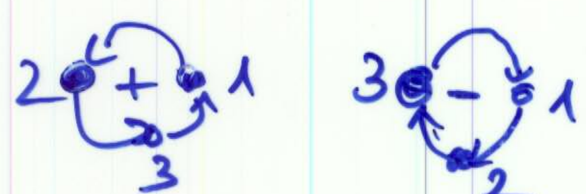
$\vec{e}_1, \vec{e}_2, \vec{e}_3$ ON, pos.

$$\vec{a} = a_1\vec{e}_1 + a_2\vec{e}_2 + a_3\vec{e}_3$$
$$\vec{b} = b_1\vec{e}_1 + b_2\vec{e}_2 + b_3\vec{e}_3$$

$$\vec{a} \times \vec{b} = \sum_{i,j=1}^{3} a_i b_1 \, \vec{e}_i \times \vec{e}_j$$

$$= a_1 b_2 \, \vec{e}_1 \times \vec{e}_2 + a_1 b_3 \, \vec{e}_1 \times \vec{e}_3$$
$$+ a_2 b_1 \, \vec{e}_2 \times \vec{e}_1 + a_2 b_3 \, \vec{e}_2 \times \vec{e}_3$$
$$+ a_3 b_1 \, \vec{e}_3 \times \vec{e}_1 + a_3 b_2 \, \vec{e}_3 \times \vec{e}_2$$

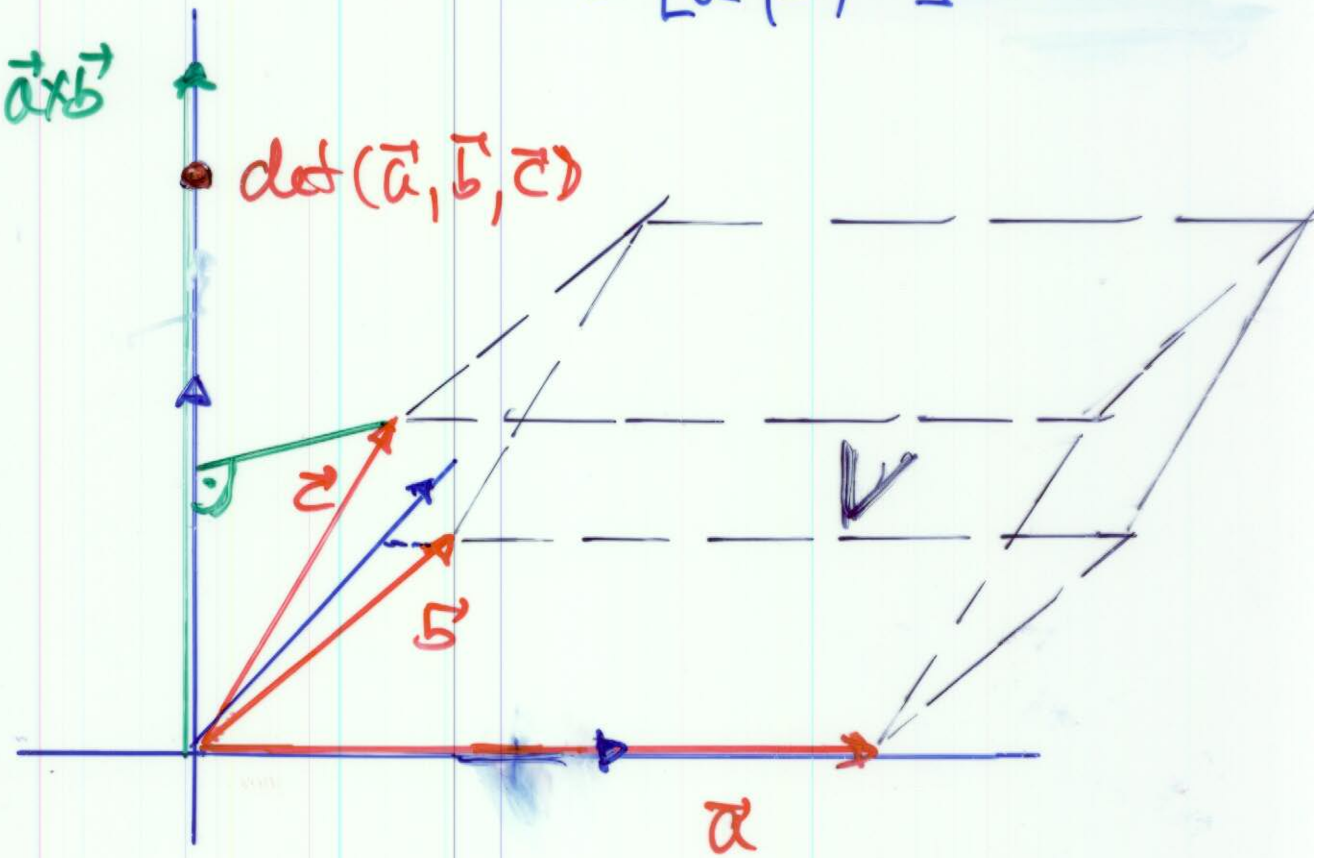
$$= (a_2 b_3 - a_3 b_2)\vec{e}_1 \qquad = \det A_1 \, \vec{e}_1$$
$$+ (a_3 b_1 - a_1 b_3)\vec{e}_2 \qquad - \det A_2 \, \vec{e}_2$$
$$+ (a_1 b_2 - a_2 b_1)\vec{e}_3 \qquad + \det A_3 \, \vec{e}_3$$



$$\begin{array}{ccccc} & & + & & \\ a_1 & b_1 & \vec{e}_1 & a_1 & b_1 \\ a_2 & b_2 & \vec{e}_2 & a_2 & b_2 \\ a_3 & b_3 & \vec{e}_3 & a_3 & b_3 \\ & - & & & \end{array}$$

Determinante, Spatprodukt

$$\det(\vec{a},\vec{b},\vec{c}) = \langle \vec{a}\times\vec{b} \mid \vec{c}\rangle$$
$$= [\vec{a},\vec{b},\vec{c}]$$



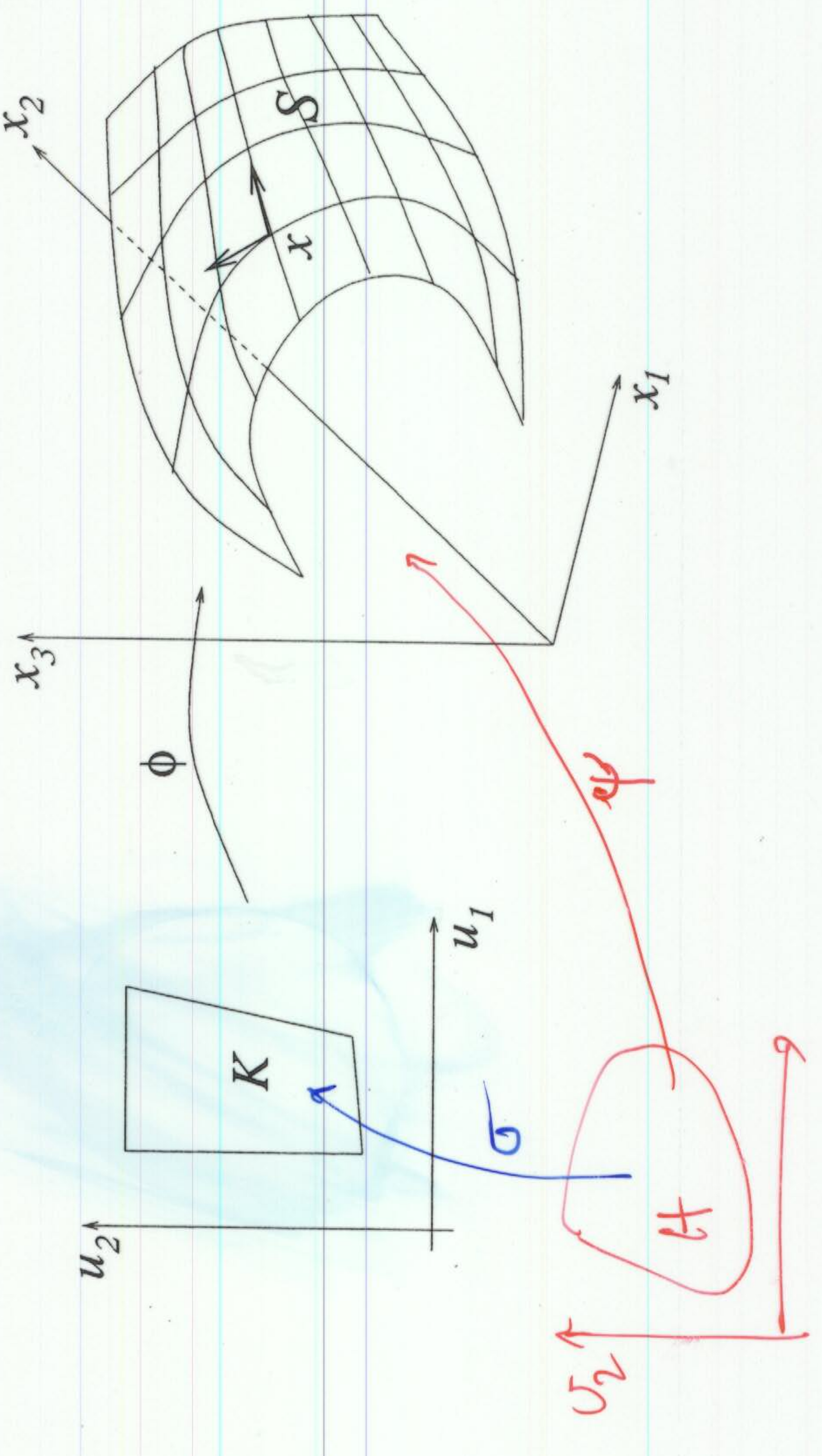
$$\det(\vec{a},\vec{b},\vec{c}) = \begin{cases} V>0 & \vec{a},\vec{b},\vec{c} \text{ pos. or.} \\ -V<0 & \vec{a},\vec{b},\vec{c} \text{ neg. or.} \\ 0 & \vec{a},\vec{b},\vec{c} \text{ abhängig} \end{cases}$$

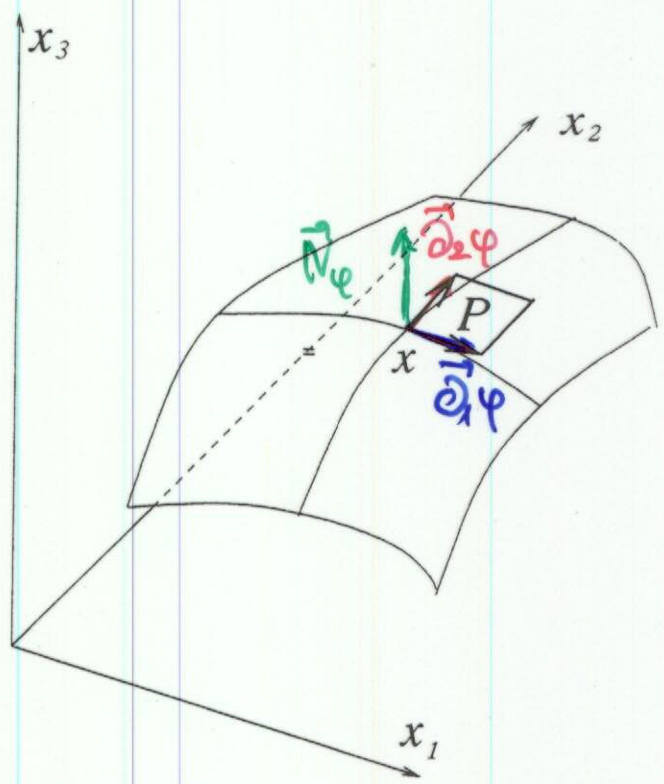
linear in $\vec{a}, \vec{b}, \vec{c}$

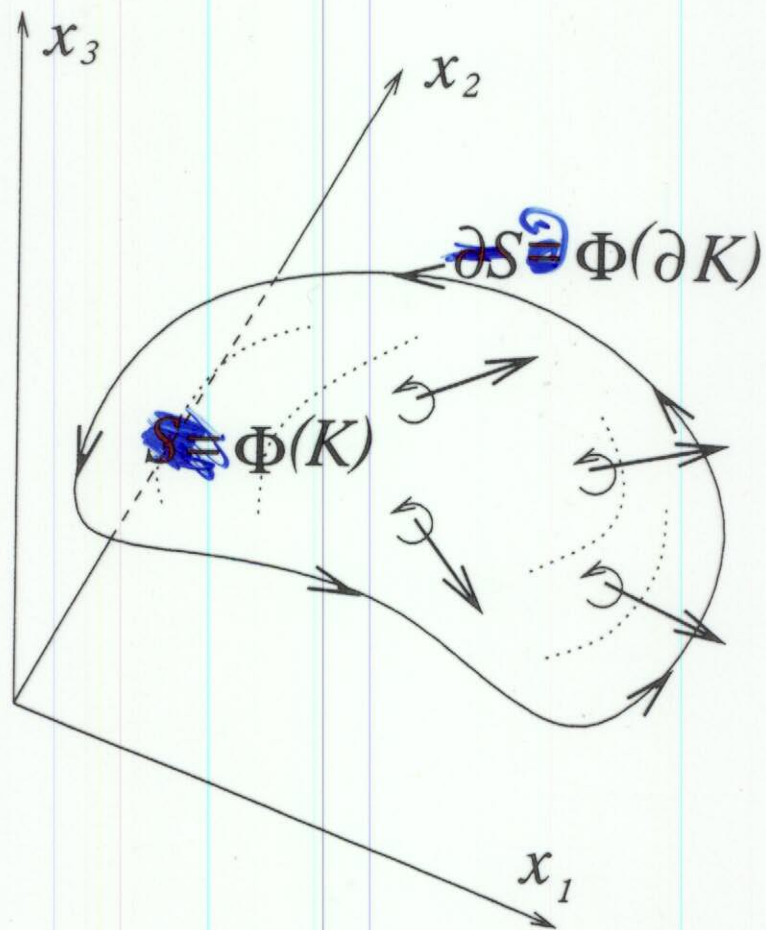
$$\det(\vec{a},\vec{a},\vec{c}) = \det(\vec{a},\vec{c},\vec{c})$$
$$= \det(\vec{c},\vec{b},\vec{c}) = 0$$

$$\det(\vec{b},\vec{a},\vec{c}) = -\det(\vec{a},\vec{b},\vec{c})$$

27.1

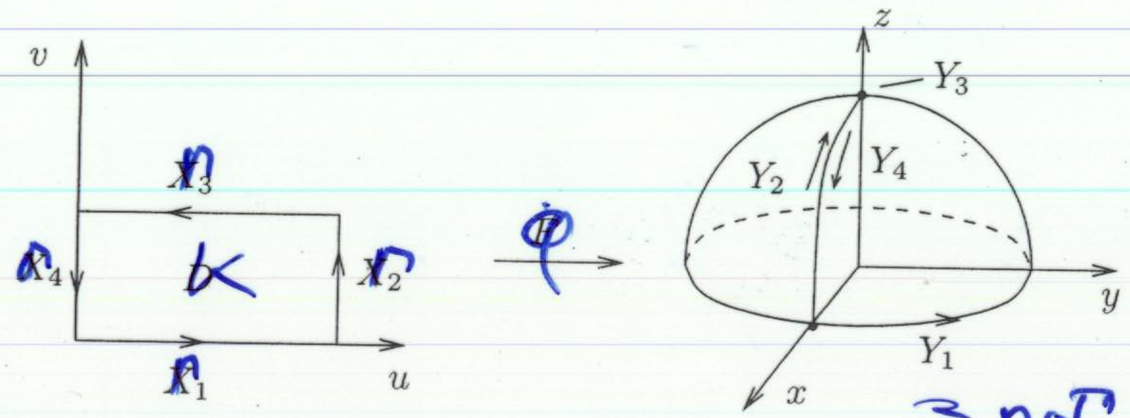



x_3
x_2
x_1
$\vec{N}_\varphi$
$\vec{\partial}_2\varphi$
P
x
$\vec{\partial}_1\varphi$


x_3
x_2
$\partial S = \Phi(\partial K)$
$S = \Phi(K)$
x_1

$$\varphi\begin{pmatrix} u \\ v \end{pmatrix} = r \begin{pmatrix} \cos u \cos v \\ \sin u \cos v \\ \sin v \end{pmatrix}$$

$$\vec{F}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ x \\ 1 \end{pmatrix}$$

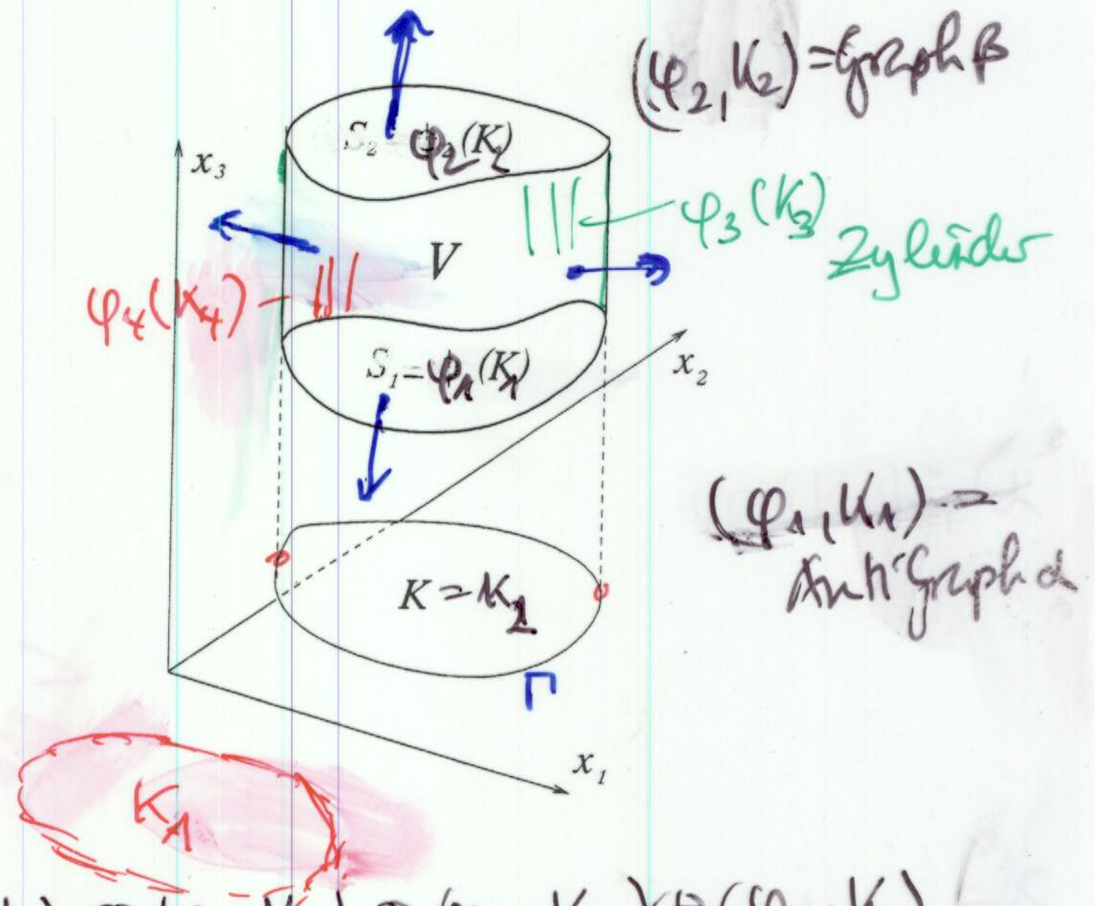


$= \varphi \circ \Gamma_1$

27.11.16

$(\Gamma, K) \subseteq$ Ebene graph

$\alpha, \beta : K \to \mathbb{R}$

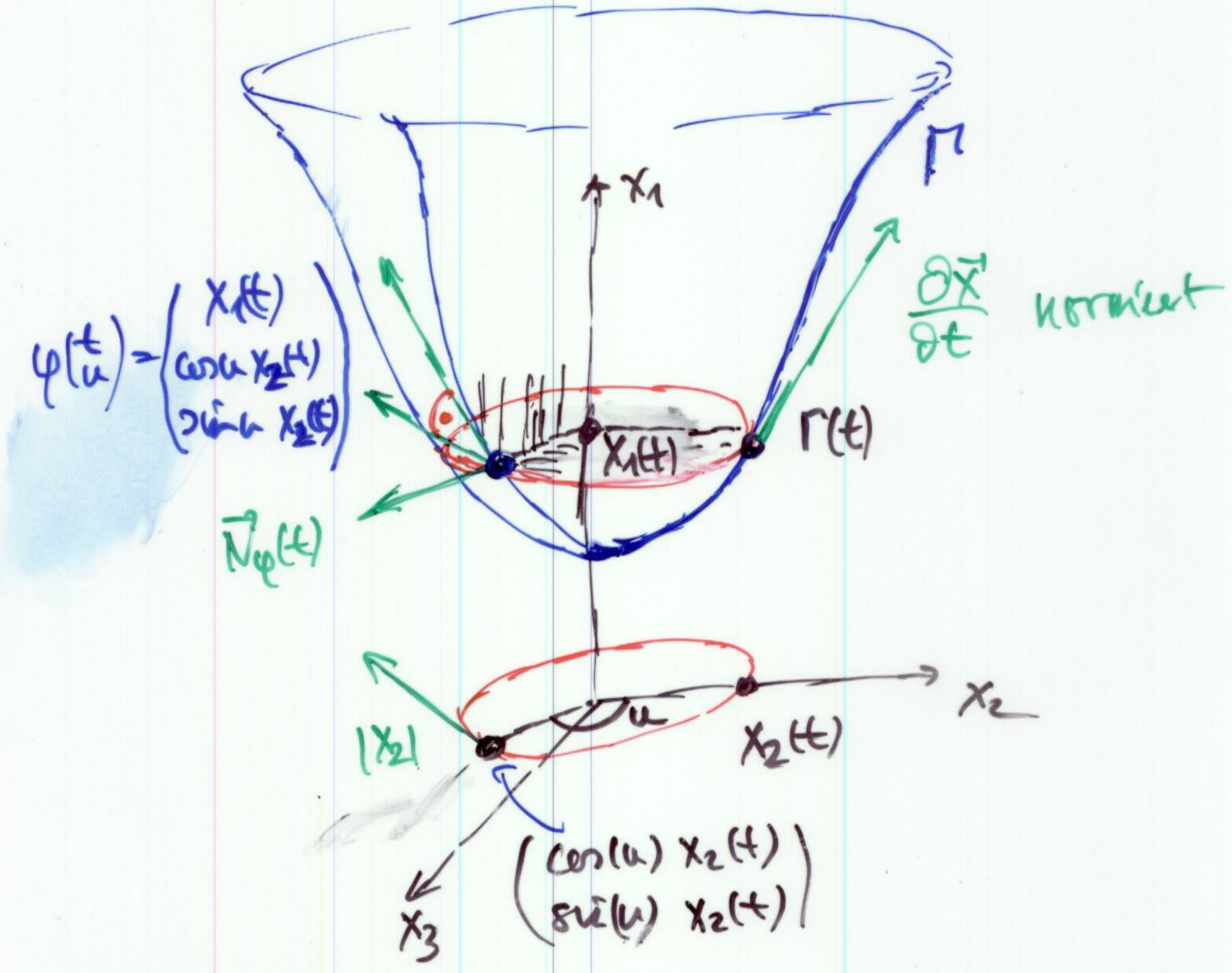


$(\varphi_1, K_1) \oplus (\varphi_2, K_2) \oplus (\varphi_3, K_3) \oplus (\varphi_4, K)$
$\in S_{12}(\Gamma, \alpha, \beta)$

$V = V_{12}(\Gamma, \alpha, \beta) = \{ \binom{u}{z} \mid u \in K \;\; \alpha u \le z \le \beta u \}$

(Γ, α, β) 1-2-graph

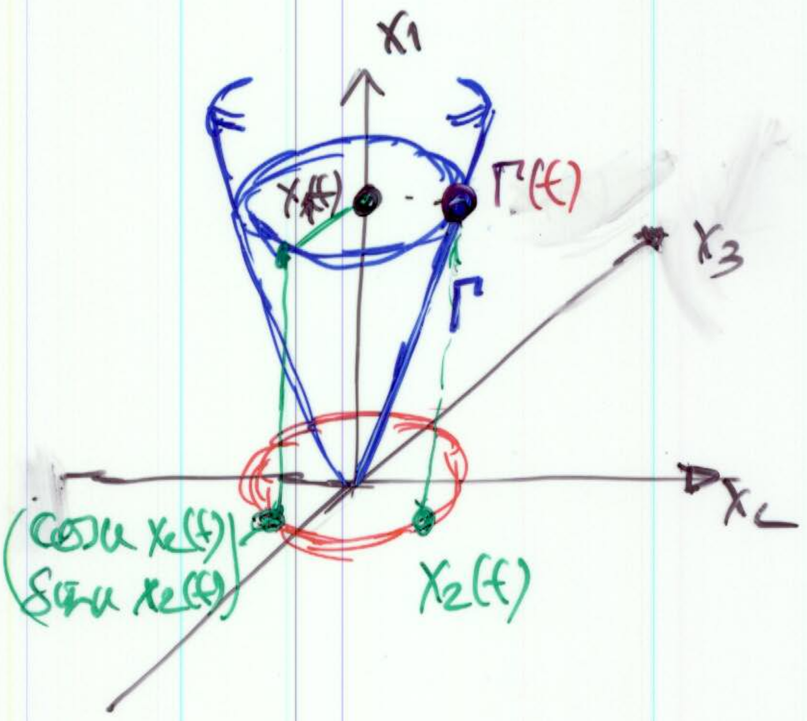
27.21 2. Guldinsche Regel



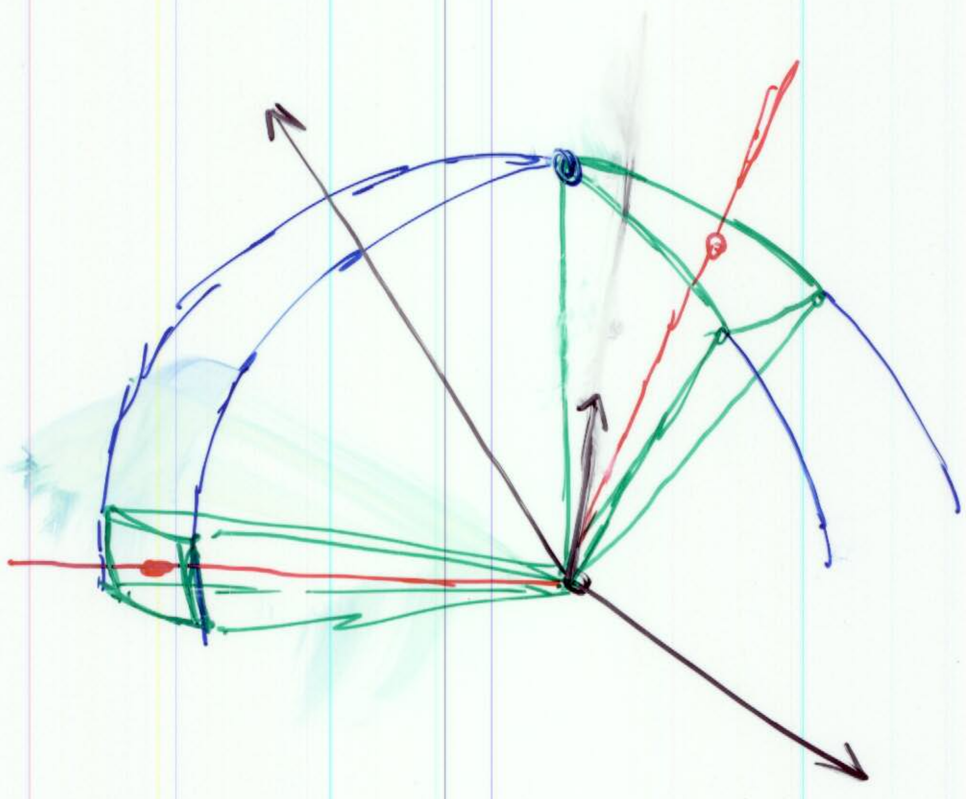
$$\left|\frac{\partial \vec{x}}{\partial t}\right| = 1 \Rightarrow \int_{(\varphi,K)} 1 = \int_0^{2\pi}\int_0^{L} x_2(t)\,dt\,du$$

$$= 2\pi L S_2$$

allgem. $$\int_{(\varphi,K)} 1 = \int_0^{2\pi}\int_a^{b} x_2(t)\left|\frac{\partial \vec{x}}{\partial t}\right| dt\,du$$

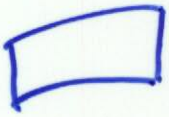


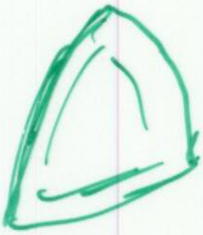
x1
X(t)
Γ(t)
x3
Γ
x2
(cos u x2(t), sin u x2(t))
x2(t)

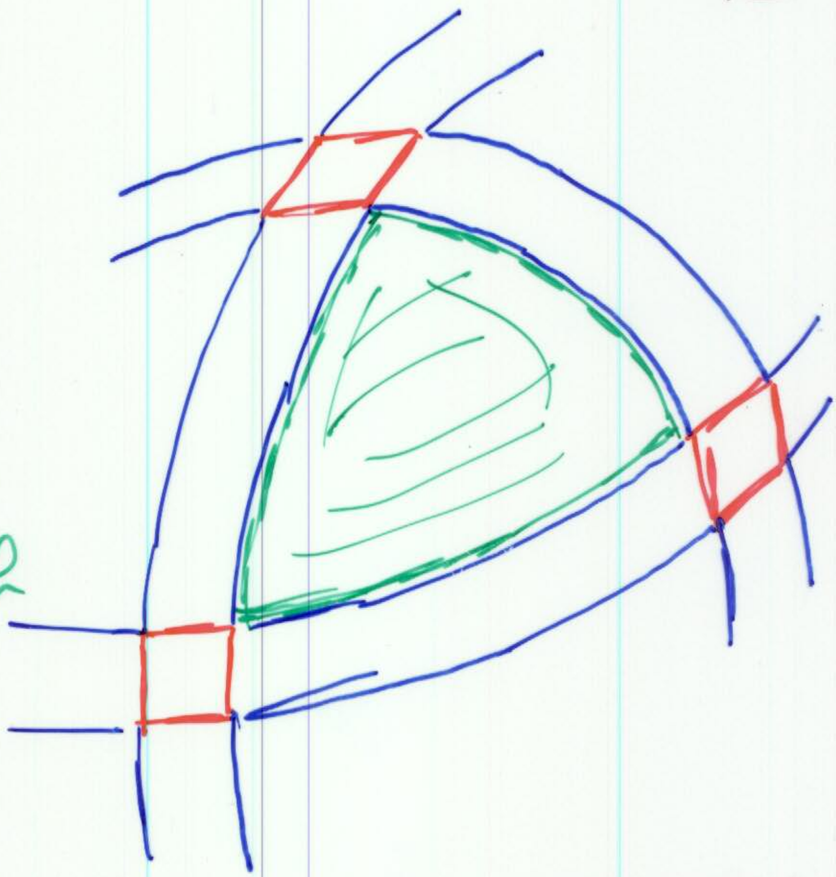


27. 11-13, 15-19


standard
eben grün
6

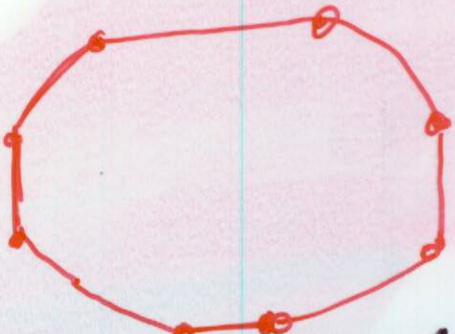

Zylinder
12


3-fach Graph
8



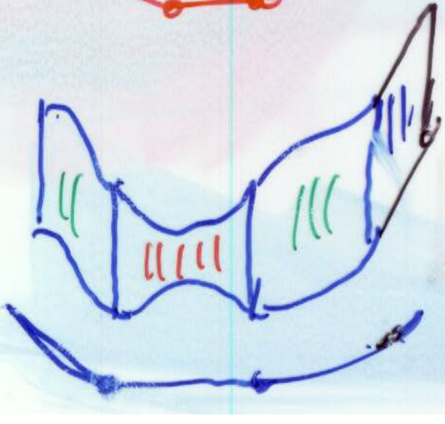
1-2 standard grün

→ 2-3 / 3-1 Zylinder



1-2 -standard Zylinder

→ ⊕ 3-1 (Anti) Graph
eben
Zylinder



$$\begin{pmatrix}\theta\\ \varphi\end{pmatrix} \mapsto \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}\cos\varphi \sin\theta\\ \sin\varphi \sin\theta\\ \cos\theta\end{pmatrix} = \psi\begin{pmatrix}\theta\\ \varphi\end{pmatrix}$$

$$J_\psi = \begin{pmatrix}\cos\varphi\cos\theta & -\sin\varphi\sin\theta\\ \sin\varphi\cos\theta & \cos\varphi\sin\theta\\ -\sin\theta & 0\end{pmatrix}$$

$$\det \pi_{y,z} \circ \psi = \cos\varphi \sin^2\theta$$

$$\det \pi_{x,z} \circ \psi = \sin\varphi \sin^2\theta$$

$$\det \pi_{x,y} \circ \psi = \cos\theta \sin\theta$$

$$\begin{pmatrix}x\\ y\end{pmatrix} \overset{\varphi}{\mapsto} \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}x\\ y\\ \beta(x,y)\end{pmatrix} \qquad J_\varphi = \begin{pmatrix}1 & 0\\ 0 & 1\\ \frac{\partial\beta}{\partial x} & \frac{\partial\beta}{\partial y}\end{pmatrix}$$

notw. $\frac{\partial\beta}{\partial x} \neq 0 \quad \frac{\partial\beta}{\partial y} \neq 0$

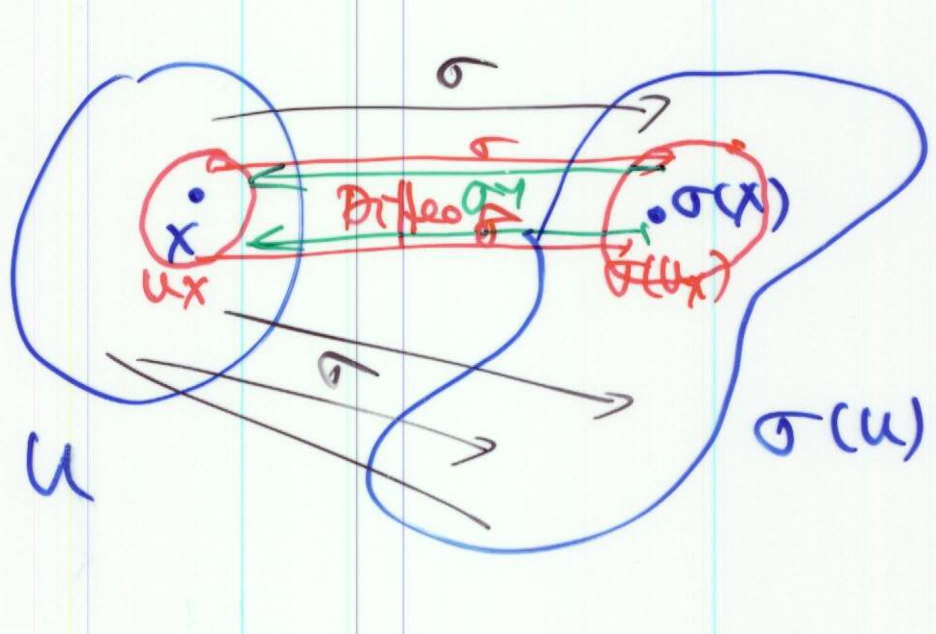
$$z = \beta(x,y)$$

27.24

Korollar zum Satz über die Umkehrfkt

$\circledast$ $\left[\begin{array}{l} \sigma : U \to \mathbb{R}^d \text{ injektiv, stetig dfftbar} \\ \operatorname{Det} \sigma > 0 \text{ auf } U \text{ oder } \operatorname{Det} \sigma < 0 \end{array}\right.$

$\Rightarrow$ $\sigma : U \to \sigma(U)$ offen
Diffeomorphismus
Homöomorphismus



25.9

Substitutionsregel

(*) $B \subseteq U$ kompakt, messbar

$\Rightarrow \sigma(B)$ messbar

$f: \sigma(B) \to \mathbb{R}$ stetig

$\Rightarrow \int_{\sigma(B)} f = \int_B f \circ \sigma \, |\det D\sigma|$

$\tau(u) = |\operatorname{Det} \sigma(u)|$

Zeige σ, τ

25.8 ist ε-Substitution

1 σ, τ stetig 2. $\sigma(B)$ messbar

3. $\exists$ Zerlegungen Z_n von B

$C \in Z_n \Rightarrow C \subseteq U$, Weite $Z_n \to 0$

(a) $\sigma(Z_n)$ Zerlegung von $\sigma(B)$

(b) Weite $\sigma(Z_n) \to 0$

(c) $\forall \varepsilon > 0 \; \exists n_0 \; \forall n \geq n_0 \; \forall C \in Z_n$

$\exists \xi_{nC} \in C$ $|\mu(\sigma(C)) - \tau(\xi_{nC})\mu(C)| \leq \varepsilon \mu(C)$

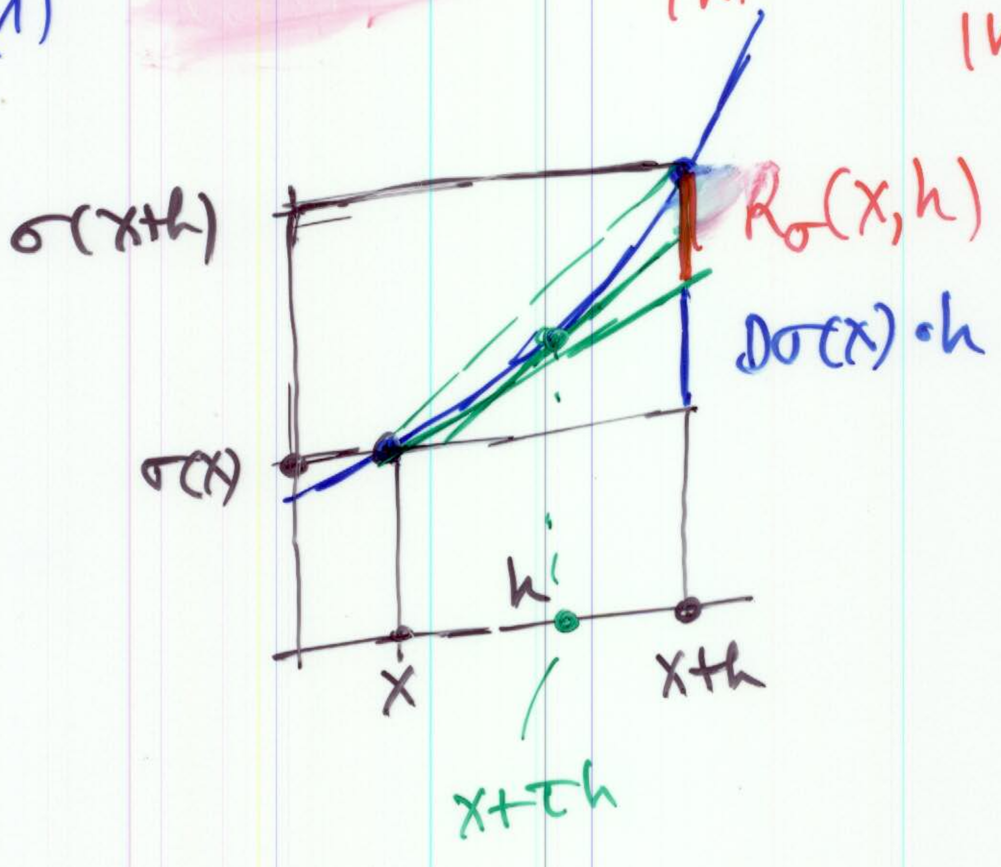
$\circledast \Rightarrow$ "alles" glm. stetig, beschränkt

J_φ, J_φ^{-1} det, Normen

28.2 Uniformer Rest

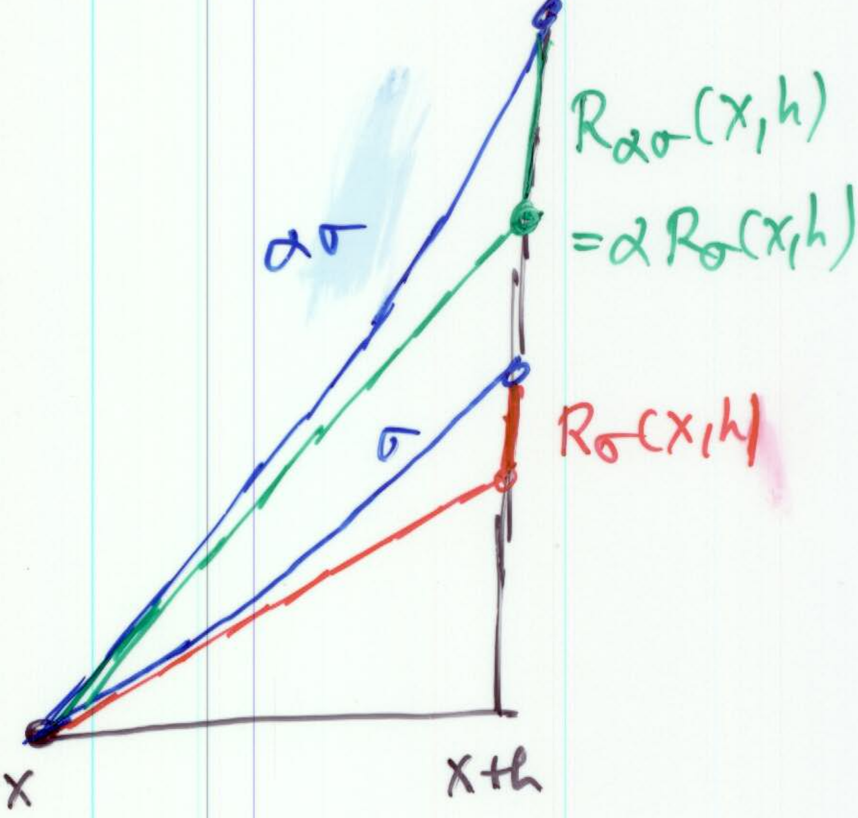
(1) $\forall \varepsilon > 0\ \exists \delta > 0 \quad \frac{|R_\sigma(x,h)|}{|h|} \leq \text{Konst} \cdot \varepsilon$

$|h| < \delta$



$$\frac{|R_\sigma(x,h)|}{|h|} \approx |D\sigma(x+\tau h) - D\sigma(x)|$$

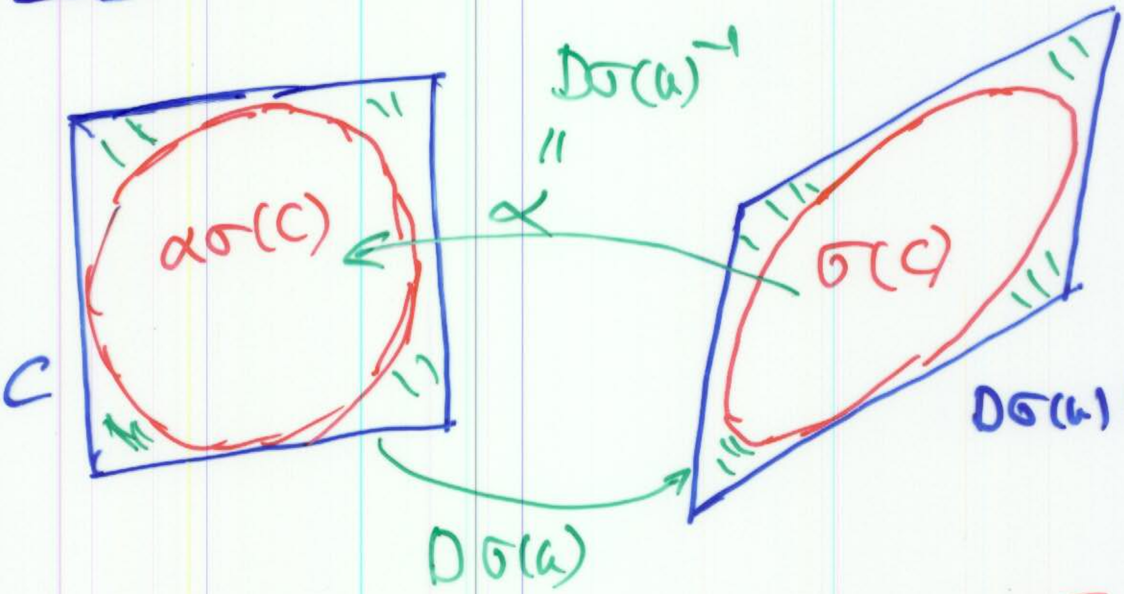
(2)



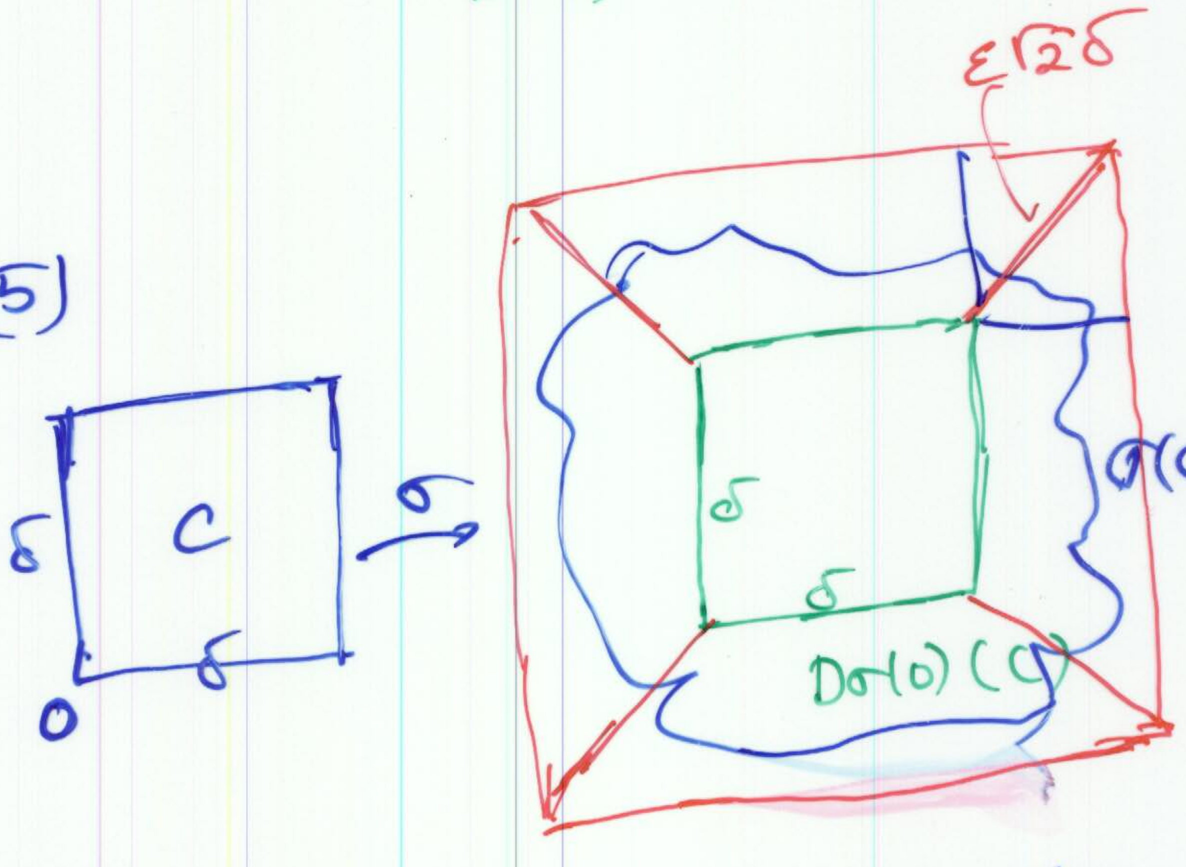
$R_{\alpha\sigma}(x,h)$
$=\alpha R_\sigma(x,h)$
$\alpha\sigma$
σ
$R_\sigma(x,h)$
x
$x+h$

28.3

(4)

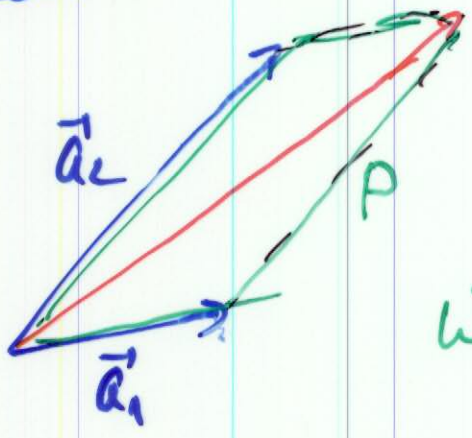


(5)



$|\sigma(h) - h| \leq |h|\,\varepsilon$

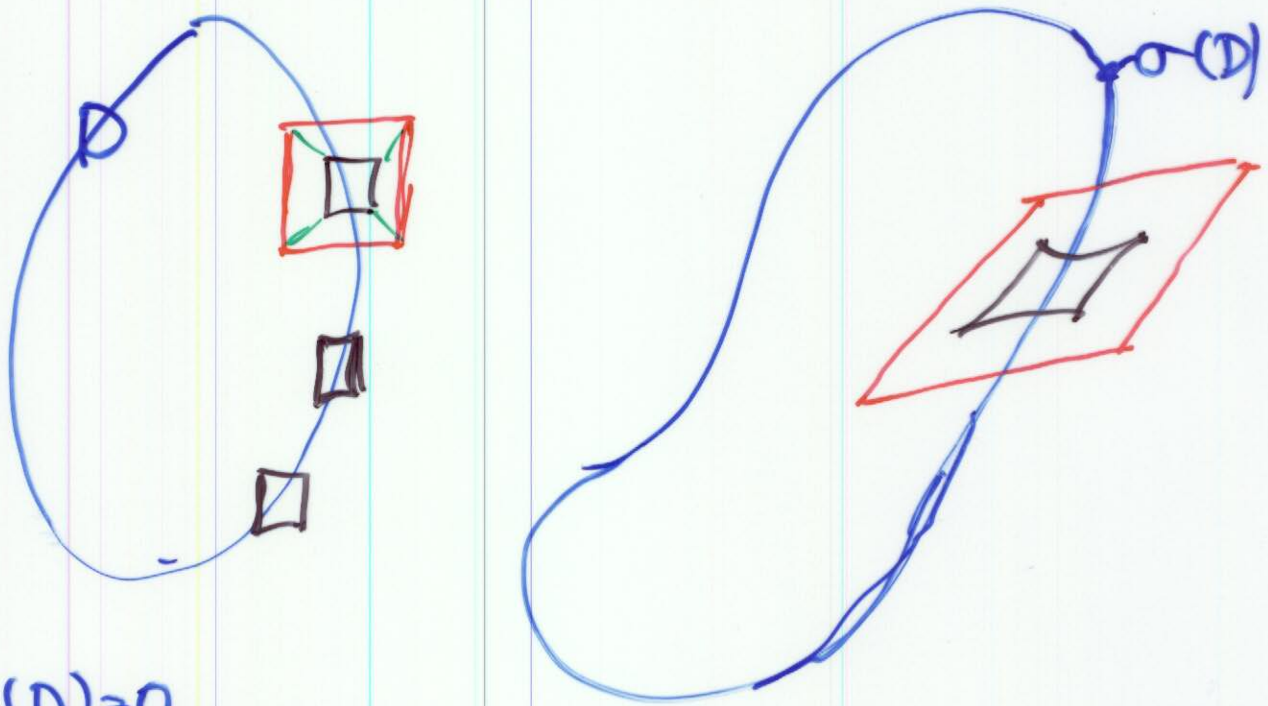
28.4



$A = (\vec{a}_1\ \vec{a}_2)$

Weite P $\leq$ dim $\cdot\ |A|$

euklid



$\mu(D) = 0$

$\sum \square < \varepsilon$

$\sum \square <$ Konst ε

$\sum \square$

$\leq \sum \square$

$= \sum \square$ det

$\leq$ Konst ε