Contents

Elementary Symbolic Commands (cont.)

Expanding an Expression

The *expand* command produces a sum of products for polynomials.
A polynomial is a mathematical expression consisting of a sum of terms each of which is a product of a constant and one or more variables with non-negative integral powers. If there is only a single variable, x, the general form is given by $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n$, where the a_i are constants (called coefficients).

Examples:

```
> restart;
> p := (x + 3)·(x − 7);
```

$$p := (x+3)\,(x-7) \tag{1}$$

```
> expand(p);
```

$$x^2 - 4\,x - 21 \tag{2}$$

```
> q := (x + 3)·(x − 7)·(x + 7); r := (x + 25)·(x − 7)·(x + 9); expand(q/r);
```

$$q := (x+3)\,(x-7)\,(x+7)$$
$$r := (x+25)\,(x-7)\,(x+9)$$
$$\frac{x^2}{(x+25)\,(x+9)} + \frac{10\,x}{(x+25)\,(x+9)} + \frac{21}{(x+25)\,(x+9)} \tag{3}$$

Factorize an Expression

The command *factor* is the opposite of the *expand* command. It factorizes polynomial exprressions.

```
> factor(x^2 − 1);
```

$$(x-1)\,(x+1) \tag{4}$$

```
> factor(%%);
```

$$\frac{(x+3)\,(x+7)}{(x+25)\,(x+9)} \tag{5}$$

Normalize fractions

Restructures rational expressions. If possible, an expression is converted to factored normal form. This is the form numerator/denominator, where the numerator and denominator are relatively prime polynomials with integer coefficients.
I.e., common factors are canceled.

> $normal\left(\frac{x^5}{x+1} + \frac{x^4}{x+1}\right);$

$$x^4 \tag{6}$$

> $normal\left(\frac{1}{x} + \frac{x}{x+1}\right);$

$$\frac{x^2 + x + 1}{x\,(x+1)} \tag{7}$$

> $normal\left(\frac{1}{x} + \frac{x}{x+1}, expanded\right);$

$$\frac{x^2 + x + 1}{x^2 + x} \tag{8}$$

> $simplify\left(\frac{x^5}{x+1} + \frac{x^4}{x+1}\right);$

$$x^4 \tag{9}$$

> $normal\left(\frac{q}{r}\right);$ *#in the output are nominator and denominator relatively prime.*

> $normal\left(\frac{q}{r}, expanded\right);$

>

Fundamental Data Structures

- fundamental data structures: expression sequences, lists, sets. (e.g. used as parameters in maple commands)

Sequences, implicitely or with command seq(f(i),i=m..n)

> 3, 5, x, 4;

$$3, 5, x, 4 \tag{10}$$

> $s := 3, 5, x, 4;$

$$s := 3, 5, x, 4 \tag{11}$$

> $s;$

$$3, 5, x, 4 \tag{12}$$

> $t := seq\left(i^2, i = 2..5\right);$

$$t := 4, 9, 16, 25 \tag{13}$$

>

A list
- is an expression sequence enclosed in square brackets
- preserves order and repetition of elements

A set
- is an expression sequence enclosed in curly brackets
- does not preserve order and does not contain the same element several times

> list1 := [5, 4, 3, 5, 4, 3];

$$list1 := [5, 4, 3, 5, 4, 3] \quad \mathbf{(14)}$$

> list2 := [3, 4, 5];

$$list2 := [3, 4, 5] \quad \mathbf{(15)}$$

> set1 := {5, 4, 3, 5, 4, 3};

$$set1 := \{3, 4, 5\} \quad \mathbf{(16)}$$

> set2 := {4, 5, 3};

$$set2 := \{3, 4, 5\} \quad \mathbf{(17)}$$

> list1[2];

$$4 \quad \mathbf{(18)}$$

Solving Equations

Example:

> restart;

> $x + y = 23$;

$$x + y = 23 \quad \mathbf{(19)}$$

> $eq1 := (x^3 - 2 \cdot x^2 + 23 \cdot x - 108 = 0)$;

$$eq1 := x^3 - 2\,x^2 + 23\,x - 108 = 0 \quad \mathbf{(20)}$$

> $eq2 := \left(2 \cdot x + 4 \cdot y = \frac{29}{6}\right)$;

$$eq2 := 2\,x + 4\,y = \frac{29}{6} \quad \mathbf{(21)}$$

> res := solve(eq1, x);

$$res := \frac{1}{3}\left(1259 + 3\sqrt{206634}\right)^{1/3} - \frac{65}{3\left(1259 + 3\sqrt{206634}\right)^{1/3}} + \frac{2}{3}, -\frac{1}{6}\left(1259 + 3\sqrt{206634}\right)^{1/3} + \frac{65}{6\left(1259 + 3\sqrt{206634}\right)^{1/3}} + \frac{2}{3} + \frac{1}{2}\,I\sqrt{3}\left(\frac{1}{3}\left(1259 + 3\sqrt{206634}\right)^{1/3} + \frac{65}{3\left(1259 + 3\sqrt{206634}\right)^{1/3}}\right), -\frac{1}{6}\left(1259 + 3\sqrt{206634}\right)^{1/3} + \frac{65}{6\left(1259 + 3\sqrt{206634}\right)^{1/3}} + \frac{2}{3} - \frac{1}{2}\,I\sqrt{3}\left(\frac{1}{3}\left(1259 + 3\sqrt{206634}\right)^{1/3}\right. \quad \mathbf{(22)}$$

$$+\frac{65}{3\left(1259+3\sqrt{206634}\right)^{1/3}}\Big)$$

> fsolve(eq1, x);

$$3.692418864 \tag{23}$$

>

> fsolve({eq1, eq2}, {x, y});

$$\{x=3.692418864,\ y=-0.6378760987\} \tag{24}$$

> solve({eq1, eq2}, {x, y});

$$\left\{x=RootOf\left(\_Z^3-2\,\_Z^2+23\,\_Z-108,\ label=\_L2\right),\ y=-\frac{1}{2}\,RootOf\left(\_Z^3-2\,\_Z^2+23\,\_Z-108,\ label=\_L2\right)+\frac{29}{24}\right\} \tag{25}$$

> evalf(res);

$$3.692418863,\ -0.8462094323+5.341633545\,\mathrm{I},\ -0.8462094323-5.341633545\,\mathrm{I} \tag{26}$$

> evalf(res[1]);

$$3.692418863 \tag{27}$$

> reslist := convert({res},'list'); # also possible: reslist := [res];

$$reslist:=\left[\frac{1}{3}\left(1259+3\sqrt{206634}\right)^{1/3}-\frac{65}{3\left(1259+3\sqrt{206634}\right)^{1/3}}+\frac{2}{3},\ -\frac{1}{6}\left(1259+3\sqrt{206634}\right)^{1/3}+\frac{65}{6\left(1259+3\sqrt{206634}\right)^{1/3}}+\frac{2}{3}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(1259+3\sqrt{206634}\right)^{1/3}+\frac{65}{3\left(1259+3\sqrt{206634}\right)^{1/3}}\right),\ -\frac{1}{6}\left(1259+3\sqrt{206634}\right)^{1/3}+\frac{65}{6\left(1259+3\sqrt{206634}\right)^{1/3}}+\frac{2}{3}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(1259+3\sqrt{206634}\right)^{1/3}+\frac{65}{3\left(1259+3\sqrt{206634}\right)^{1/3}}\right)\right] \tag{28}$$

> reslist2 := [res];

$$reslist2:=\left[\frac{1}{3}\left(1259+3\sqrt{206634}\right)^{1/3}-\frac{65}{3\left(1259+3\sqrt{206634}\right)^{1/3}}+\frac{2}{3},\ -\frac{1}{6}\left(1259+3\sqrt{206634}\right)^{1/3}+\frac{65}{6\left(1259+3\sqrt{206634}\right)^{1/3}}+\frac{2}{3}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(1259+3\sqrt{206634}\right)^{1/3}+\frac{65}{3\left(1259+3\sqrt{206634}\right)^{1/3}}\right),\ -\frac{1}{6}\left(1259+3\sqrt{206634}\right)^{1/3}+\frac{65}{6\left(1259+3\sqrt{206634}\right)^{1/3}}+\frac{2}{3}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(1259+3\sqrt{206634}\right)^{1/3}\right. \tag{29}$$

$$+ \frac{65}{3\left(1259+3\sqrt{206634}\right)^{1/3}}\Big)\Big]$$

> *evalf*(*reslist*[1]);

>

> *solve*({*x* = *reslist*[1], *eq2*}, {*x*, *y*});

> *fsolve*({*x* = *reslist*[1], *eq2*}, {*x*, *y*});

>

Another example:

> $\frac{\text{factorial}(10000)}{\text{factorial}(9999)}$;

$$10000 \quad \mathbf{(30)}$$

> *solve*(*x*·factorial(10000) = 10000, *x*);

1 / **(31)**

2846259680917054518906413212119868890148051401702799230794179994274411340\
03764443772990786757784775[...35456 digits...]
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000

Differential- und Integralrechnung

> restart;

> $f := x^5 + x^2 - 3 \cdot x + 2;$

$$f := x^5 + x^2 - 3\,x + 2 \quad \mathbf{(32)}$$

> $f1_{inert} := Diff(f, x);$

$$f1_{inert} := \frac{d}{dx}\left(x^5 + x^2 - 3\,x + 2\right) \quad \mathbf{(33)}$$

> $f1 := diff(f, x);$

$$f1 := 5\,x^4 + 2\,x - 3 \quad \mathbf{(34)}$$

> *solve*($f1 = 0, x$);

$$-1, \frac{1}{3}10^{1/3} - \frac{1}{15}10^{2/3} + \frac{1}{3}, -\frac{1}{6}10^{1/3} + \frac{1}{30}10^{2/3} + \frac{1}{3} + \frac{1}{2}I\sqrt{3}\left(\frac{1}{3}10^{1/3} + \frac{1}{15}10^{2/3}\right), -\frac{1}{6}10^{1/3} + \frac{1}{30}10^{2/3} + \frac{1}{3} - \frac{1}{2}I\sqrt{3}\left(\frac{1}{3}10^{1/3} + \frac{1}{15}10^{2/3}\right) \quad \mathbf{(35)}$$

> *eval*$\left(f1_{inert}, x = solve(f1 = 0, x)[2]\right)$;
#puts the value into the inert-function. simplify does not proceed.

$$\left(5\left(\frac{1}{3}10^{1/3} - \frac{1}{15}10^{2/3} + \frac{1}{3}\right)^4 + \frac{2}{3}10^{1/3} - \frac{2}{15}10^{2/3} - \frac{7}{3}\right)_{inert} \quad \mathbf{(36)}$$

> $eval\left(5\cdot x^4 + 2\cdot x - 3, x = 1\right);$

$$4 \quad \textbf{(37)}$$

> $f2_{inert} := Int\left(x^5 + x^2 - 3\cdot x + 2, x\right);$

$$f2_{inert} := \int \left(x^5 + x^2 - 3\,x + 2\right) \mathrm{d}x \quad \textbf{(38)}$$

> $f2 := int\left(x^5 + x^2 - 3\cdot x + 2, x\right); f2 := int(f, x);$

$$f2 := \frac{1}{6}\,x^6 + \frac{1}{3}\,x^3 - \frac{3}{2}\,x^2 + 2\,x$$

$$f2 := \frac{1}{6}\,x^6 + \frac{1}{3}\,x^3 - \frac{3}{2}\,x^2 + 2\,x \quad \textbf{(39)}$$

> $f2tan25 := eval\left(f, x = \frac{5}{2}\right); unapply(diff(f2, x), x)\left(\frac{5}{2}\right);$

$$f2tan25 := \frac{3149}{32}$$

$$\frac{3149}{32} \quad \textbf{(40)}$$

> $ffunc := unapply(f, x); f2func := unapply(f2, x);$

$$ffunc := x \rightarrow x^5 + x^2 - 3\,x + 2$$

$$f2func := x \rightarrow \frac{1}{6}\,x^6 + \frac{1}{3}\,x^3 - \frac{3}{2}\,x^2 + 2\,x \quad \textbf{(41)}$$

> $unassign('b'); b := solve\left(f2tan25\cdot\frac{5}{2} + b = f2func\left(\frac{5}{2}\right), b\right);$

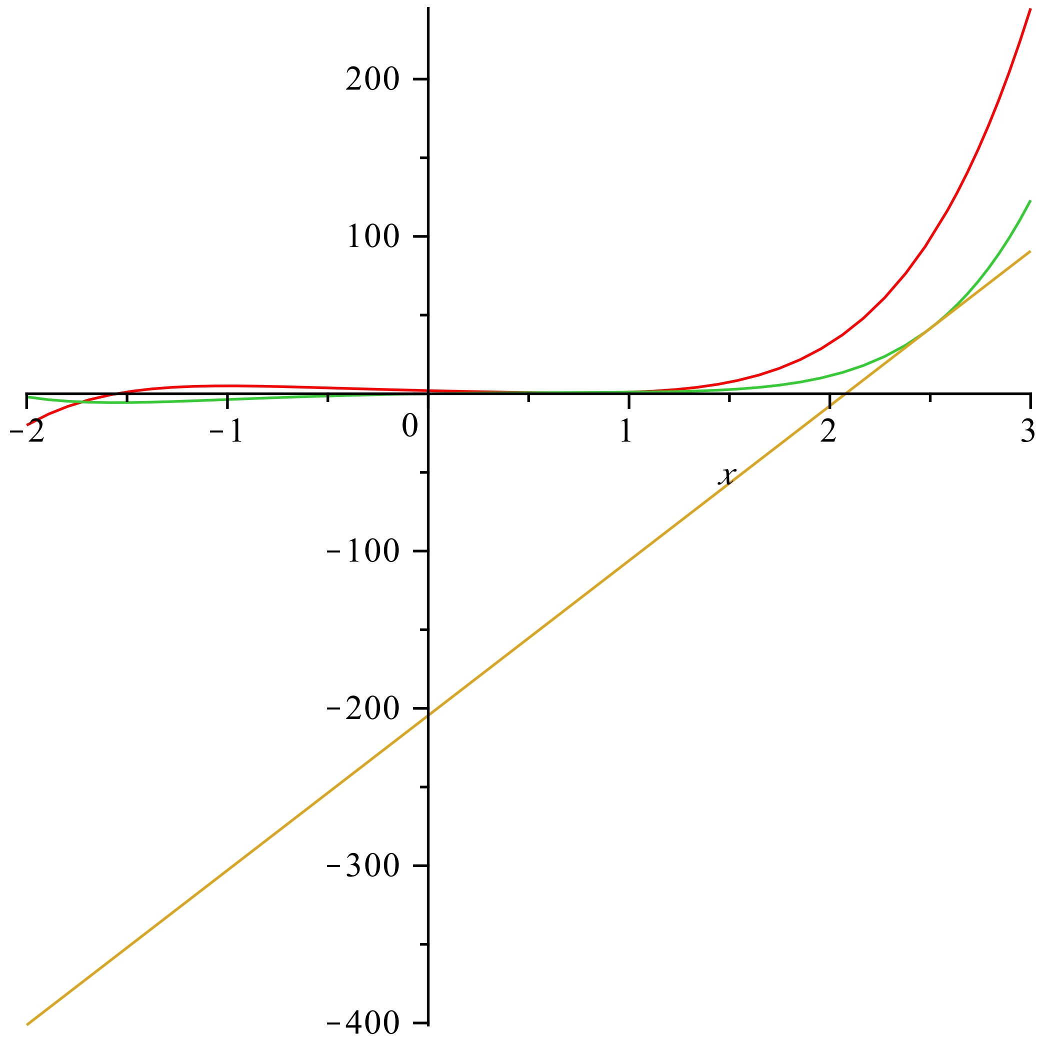
$$b := -\frac{26175}{128} \quad \textbf{(42)}$$

> $ffunc\left(\frac{5}{2}\right); eval\left(f2, x = \frac{5}{2}\right);$

$$\frac{3149}{32}$$

$$\frac{5315}{128} \quad \textbf{(43)}$$

> $plot([f, f2, x\cdot f2tan25 + b], x = -2..3);$



>

> *ffunc*(2 + 1);

> *ffunc*(2 + 1);

> 2·3; *x y*;

$$6$$
$$x\,y \qquad \textbf{(44)}$$

> *x* := 2 ; *y* := 3; *x* (*y* + 1); *x*· (*y* + 1);

$$x := 2$$
$$y := 3$$
$$8$$
$$8 \qquad \textbf{(45)}$$

> *unassign*('*x*'); *unassign*('*y*');

The Maple Libraries

The Maple library consists of for parts:

• the standard library
• the update library
• packages
• share library (user-contributed)

Until now, we only used commands and operations from the standard andthe update library.

Howver: There are so called packeges for more specialized purposes in Maple,
e.g. the LinearAlgebra package for matrx-vector computations or the numtheory-package.
Functions from those packages can be used with the following syntax:

PackageName[FunctionName](FunctionParameters)

Here two examples:

```
>
>
> A := Matrix([[1, 2], [3, 4]]);
```

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \textbf{(46)}$$

```
> LinearAlgebra[Transpose](A); # transposes the matrix A
```

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad \textbf{(47)}$$

```
> numtheory[divisors](68); # prints the divisors of 68 to the screen
```

$$\{1, 2, 4, 17, 34, 68\} \quad \textbf{(48)}$$

Often, you want to use a package more intensively. Then you can abbriviate the package-commands with the with()-command:

```
> with(LinearAlgebra) :
> with(numtheory);
```

[GIgcd, bigomega, cfrac, cfracpol, cyclotomic, divisors, factorEQ, factorset, fermat, imagunit, index, integral_basis, invcfrac, invphi, iscyclotomic, issqrfree, ithrational, jacobi, kronecker, λ, legendre, mcombine, mersenne, migcdex, minkowski, mipolys, mlog, mobius, mroot, msqrt, nearestp, nthconver, nthdenom, nthnumer, nthpow, order, pdexpand, ϕ, π, pprimroot, primroot, quadres, rootsunity, safeprime, σ, sq2factor, sum2sqr, τ, thue] **(49)**

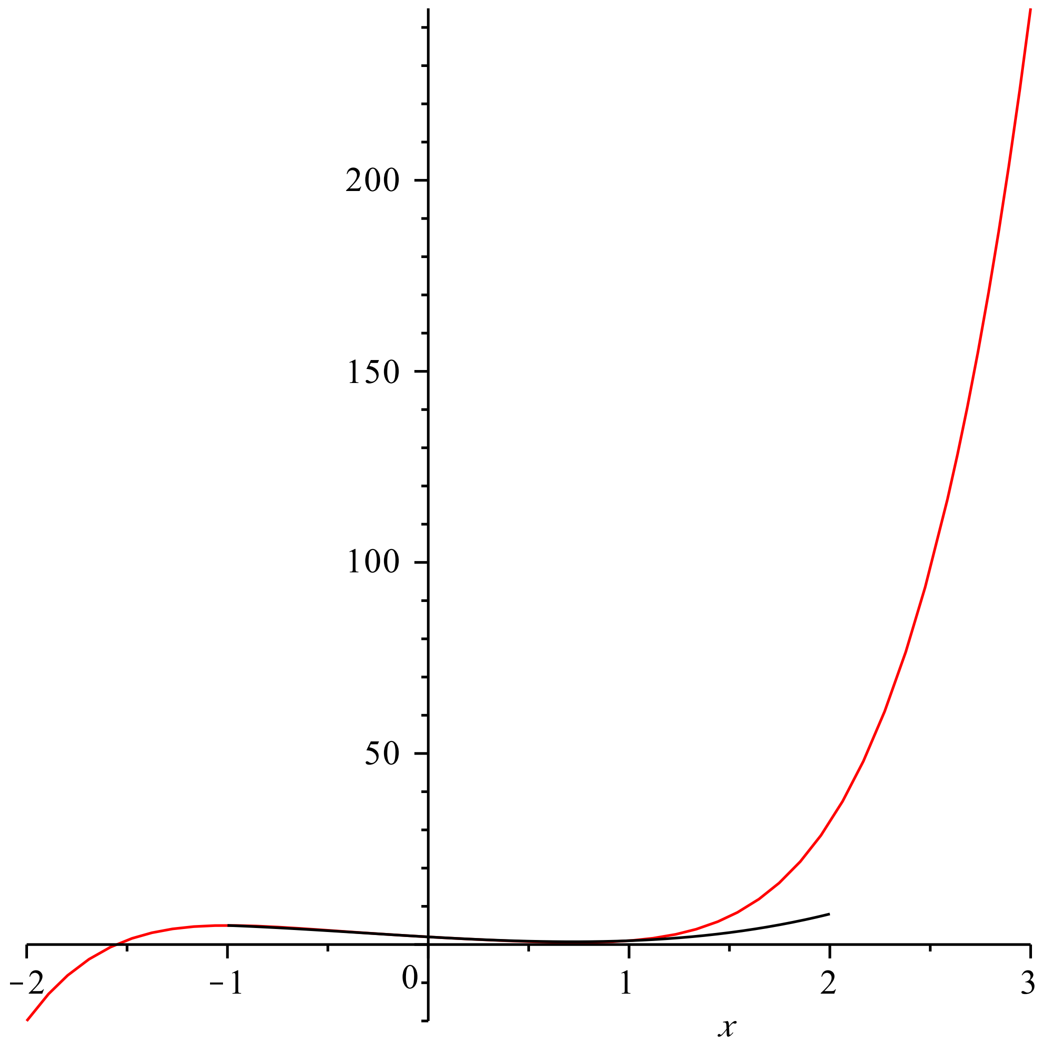
```
> A, Transpose(A);
```

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad \textbf{(50)}$$

```
> factorset(96);
                                   {2, 3}                                   (51)
```

Points, Vectors, and Maztrices

```
> restart; with(LinearAlgebra);
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,   (52)
    BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
    ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
    ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation,
    CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix,
    Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors,
    Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations,
    GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix,
    GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm,
    HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite,
    IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct,
    LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2,
    MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply,
    MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply,
    MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace,
    OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix,
    RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row,
    RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector,
    SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector,
    SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose,
    TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle,
    VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]
> f := x→x^5 + x^2 − 3·x + 2; pp := plot(f(x), x=-2..3); ppp := plot(x^3 + x^2 − 3·x
      + 2, x=-1..2, color = black);
                           f := x→x^5 + x^2 − 3 x + 2
                                pp := PLOT(...)
                               ppp := PLOT(...)                             (53)
> display(pp);
                              display(PLOT(...))                            (54)
> with(plots):
> display(pp, ppp);
```



Let us inspect (column) vectors.

> $p := \langle 0, 1 \rangle; r := \langle 1, 2 \rangle;$

$$p := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$r := \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \textbf{(55)}$$

> $p[1];$

$$0 \quad \textbf{(56)}$$

> $r[2];$

$$2 \quad \textbf{(57)}$$

> $p + r;$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \textbf{(58)}$$

> $l := p + \lambda \cdot r;$

$$l := \begin{bmatrix} \lambda \\ 1 + 2\lambda \end{bmatrix} \quad \mathbf{(59)}$$

Now, we want to compute the shortest distance from point q := <2,1> to the line.

> $q := \langle 2, 1 \rangle;$

$$q := \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \mathbf{(60)}$$

> $lineplot := plot([l[1], l[2], \lambda = -2..2]);$

$$lineplot := PLOT(...) \quad \mathbf{(61)}$$

> $f := \lambda \rightarrow p + \lambda \cdot r;$

$$f := \lambda \rightarrow p + \lambda\, r \quad \mathbf{(62)}$$

> $s := seq([l[1], l[2]], \lambda = -2..2);$

$$s := [-2, -3], [-1, -1], [0, 1], [1, 3], [2, 5] \quad \mathbf{(63)}$$

> $t := seq\left(\left[f\left(\frac{x}{10}\right)[1], f\left(\frac{x}{10}\right)[2]\right], x = -20..20\right);$

$$t := [-2, -3], \left[-\frac{19}{10}, -\frac{14}{5}\right], \left[-\frac{9}{5}, -\frac{13}{5}\right], \left[-\frac{17}{10}, -\frac{12}{5}\right], \left[-\frac{8}{5}, -\frac{11}{5}\right], \left[-\frac{3}{2}, -2\right], \left[-\frac{7}{5}, -\frac{9}{5}\right], \left[-\frac{13}{10}, -\frac{8}{5}\right], \left[-\frac{6}{5}, -\frac{7}{5}\right], \left[-\frac{11}{10}, -\frac{6}{5}\right], [-1, -1], \left[-\frac{9}{10}, -\frac{4}{5}\right], \left[-\frac{4}{5}, -\frac{3}{5}\right], \left[-\frac{7}{10}, -\frac{2}{5}\right], \left[-\frac{3}{5}, -\frac{1}{5}\right], \left[-\frac{1}{2}, 0\right], \left[-\frac{2}{5}, \frac{1}{5}\right], \left[-\frac{3}{10}, \frac{2}{5}\right], \left[-\frac{1}{5}, \frac{3}{5}\right], \left[-\frac{1}{10}, \frac{4}{5}\right], [0, 1], \left[\frac{1}{10}, \frac{6}{5}\right], \left[\frac{1}{5}, \frac{7}{5}\right], \left[\frac{3}{10}, \frac{8}{5}\right], \left[\frac{2}{5}, \frac{9}{5}\right], \left[\frac{1}{2}, 2\right], \left[\frac{3}{5}, \frac{11}{5}\right], \left[\frac{7}{10}, \frac{12}{5}\right], \left[\frac{4}{5}, \frac{13}{5}\right], \left[\frac{9}{10}, \frac{14}{5}\right], [1, 3], \left[\frac{11}{10}, \frac{16}{5}\right], \left[\frac{6}{5}, \frac{17}{5}\right], \left[\frac{13}{10}, \frac{18}{5}\right], \left[\frac{7}{5}, \frac{19}{5}\right], \left[\frac{3}{2}, 4\right], \left[\frac{8}{5}, \frac{21}{5}\right], \left[\frac{17}{10}, \frac{22}{5}\right], \left[\frac{9}{5}, \frac{23}{5}\right], \left[\frac{19}{10}, \frac{24}{5}\right], [2, 5] \quad \mathbf{(64)}$$

> $pointline := pointplot([s]);$

$$pointline := PLOT(...) \quad \mathbf{(65)}$$

> $Qplot := pointplot(q);$

$$Qplot := PLOT(...) \quad \mathbf{(66)}$$

> $a1 := arrow([0, 0], p, width = [0.03, relative = true], head\_length = [0.2, relative = false], color = blue);$

$$a1 := PLOT(...) \quad \mathbf{(67)}$$

>

> $a2 := arrow(p, 0.25 \cdot r, width = [0.08, relative = true], head\_length = [0.2, relative = false], color = blue);$

$$a2 := PLOT(...) \quad \mathbf{(68)}$$

> $aHitQ := arrow\left(\left\langle \frac{2}{5}, \frac{9}{5} \right\rangle, q - \left\langle \frac{2}{5}, \frac{9}{5} \right\rangle, width = [0.0125, relative = true], head\_length = [0.1, relative = false], color = green\right);$

$$aHitQ := PLOT(\ldots) \tag{69}$$

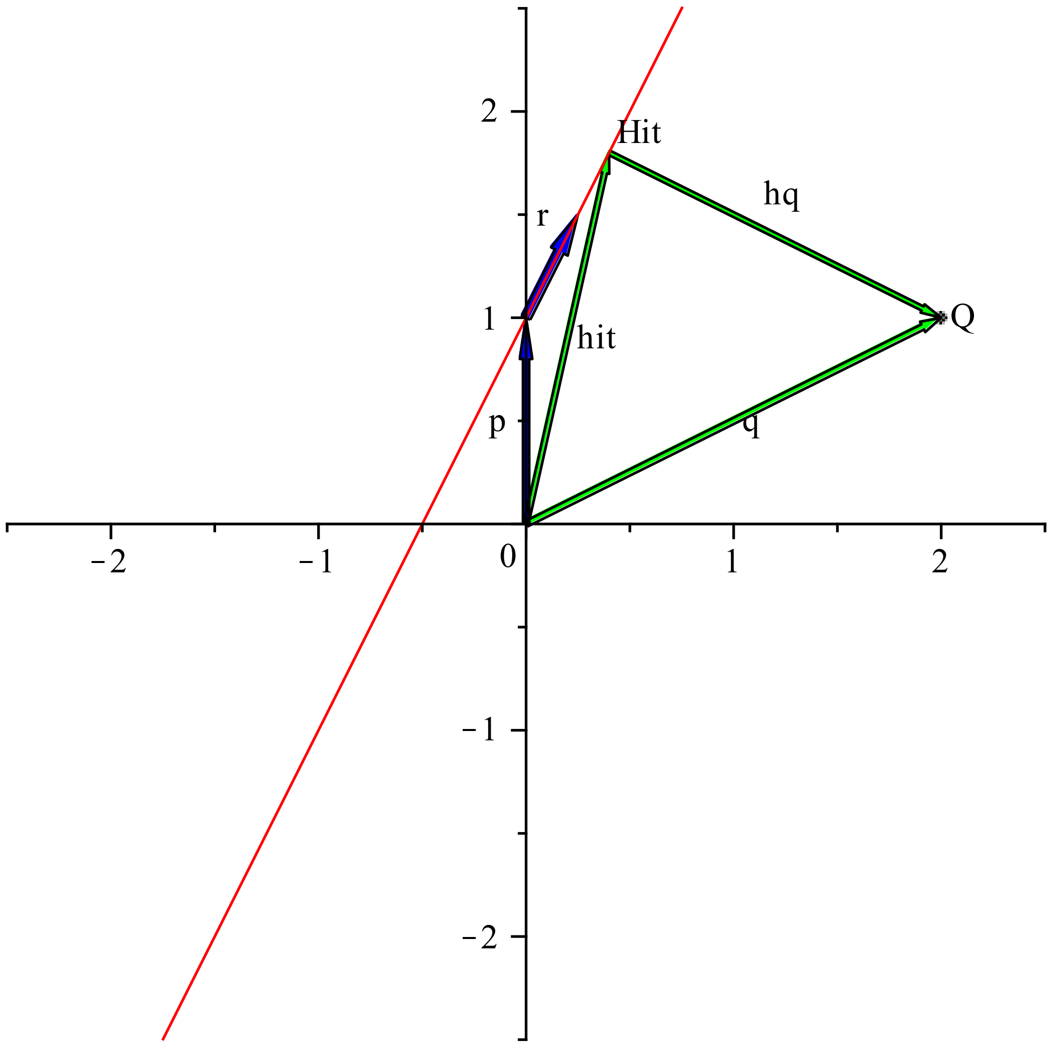
> $aQ := arrow(\langle 0, 0\rangle, q, width = [0.0125, relative = true], head\_length = [0.1, relative = false], color = green);$

$$aQ := PLOT(\ldots) \tag{70}$$

> $aHit := arrow\left(\langle 0, 0\rangle, \left\langle \frac{2}{5}, \frac{9}{5} \right\rangle, width = [0.0125, relative = true], head\_length = [0.1, relative = false], color = green\right);$

$$aHit := PLOT(\ldots) \tag{71}$$

> $display\left(a1, a2, aHit, aQ, aHitQ, Qplot, lineplot, view = [-2.5..2.5, -2.5..2.5], textplot([q[1], q[2], \text{"Q"}], align = \{right\}), textplot([1.1, 1.5, \text{"hq"}], align = \{right, above\}), textplot\left(\left[\frac{1}{5}, \frac{4}{5}, \text{"hit"}\right], align = \{right, above\}\right), textplot([1, 0.4, \text{"q"}], align = \{right, above\}), textplot\left(\left[\frac{2}{5}, \frac{9}{5}, \text{"Hit"}\right], align = \{right, above\}\right), textplot([-.02, 0.5, \text{"p"}], align = \{left\}), textplot([0.1, 1.4, \text{"r"}], align = \{above\})\right);$



>

Remarks:
q = hit+hq, thus hq=q-hit
moreover there is a lambda such that: hit = p + λr, thus hq = q - (p+λr)
now, we demand that hq ⊥ r, thus hq·r = 0, and therefore (q - (p+λr)) · r = 0

Which λ does it? And what is the Point "Hit"?

> $\lambda := solve((q - (p + x \cdot r)).r = 0, x); p + r \cdot \lambda; f\left(\frac{2}{5}\right);$

$$\lambda := \frac{2}{5}$$

$$\left[\begin{array}{c} \frac{2}{5} \\ \frac{9}{5} \end{array}\right]$$

$$\begin{bmatrix} \frac{2}{5} \\ \frac{9}{5} \end{bmatrix} \quad \textbf{(72)}$$

Remarks:
resorting (q - (p+λr)) · r = 0 leads to
(q - p - λ·r) . r = 0 and
q.r-p.r = λ·r.r and thus to $\lambda = \frac{(q-p).r}{r.r}$

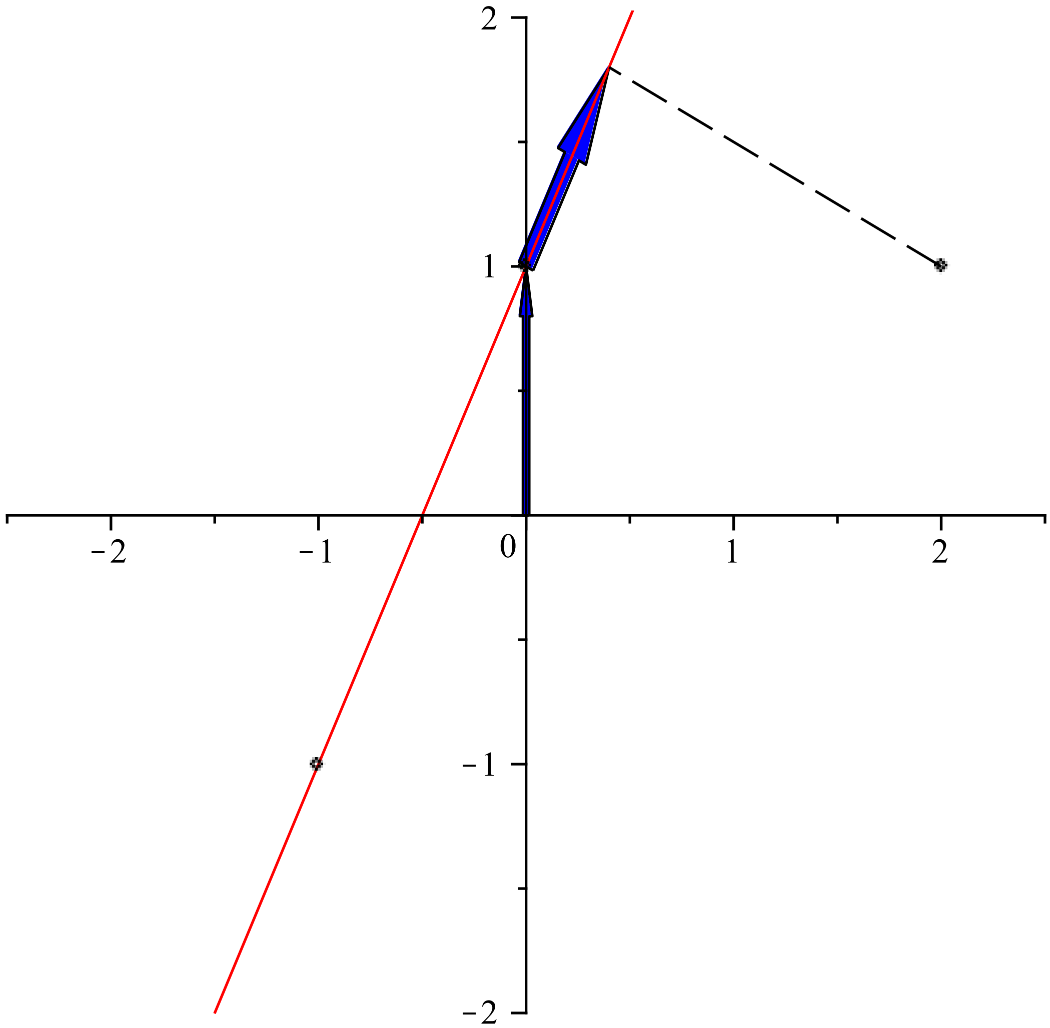
> $a22 := arrow\left(p, \frac{(q-p).r}{r.r} \cdot r, width = [0.075, relative = false], head\_length = [0.4, relative = false], color = blue\right);$

$$a22 := PLOT(\ldots) \quad \textbf{(73)}$$

> $HitPoint := subs\left(\lambda = \frac{DotProduct(q-p, r)}{DotProduct(r, r)}, l\right);$

$$HitPoint := \begin{bmatrix} \frac{2}{5} \\ \frac{9}{5} \end{bmatrix} \quad \textbf{(74)}$$

> $display([a1, a22, Qplot, lineplot, pointline, pointplot([q, HitPoint], connect = true, thickness = 1, linestyle = dash)], view = [-2.5..2.5, -2..2]);$



> *# at the end, we could plot the big arrow from (0,1), pointing exactly into the HitPoint*

>

> $\frac{DotProduct(q-p, r)}{DotProduct(r, r)}, \frac{(q-p).r}{r.r};$

>