Vorlesung

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Übungen

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Mo.	08:00 bis 09:40	ca. 14-tägig	ab 25.10.2010	S2|15 K313, K309
Di.	08:00 bis 09:40	ca. 14-tägig	ab 26.10.2010	S2|15 K313, K309
Mi.	16:15 bis 17:55	ca. 14-tägig	ab 27.10.2010	S2|15 K313, K309
Do.	09:50 bis 11:30	ca. 14-tägig	ab 28.10.2010	S2|15 K313, K309
Fr.	09:50 bis 11:30	ca. 14-tägig	ab 29.10.2010	S2|15 K313, K309

Literatur

- Es gibt Begleitmaterial (Maplesheet).
- Es wird an einem Skript/Buch gearbeitet.
- Skript zu "Introductory Course Mathematics"
  https://www3.mathematik.tu-darmstadt.de/index.php?id=84&evsid=23&&evsver=910&evsdir=868

Prüfung

- Testataufgaben mit Maple, zum Ende der Vorlesung
- 3 ECTS, 1V + 1Ü, Schwerpunkt auf Übungen in Kleingruppen

Modulhandbuch

Matrixarithmetik und lineare Gleichungssysteme, Unterschiede zwischen symbolischem und numerischem Rechnen, Differenzieren und Integrieren, Grenzwerte und Reihen, Graphik und Visualisierung, Definition von Funktionen und Programmierung

Übungsgruppen

- Listeneintragung im EVS
- Beginn der Übungen: ab Mo dem 25.10.2010
- Web-Seite der Vorlesung:
  https://www3.mathematik.tu-darmstadt.de/evs/924

Übungsaufgaben/Übungsbetrieb

- Ausgabe der Übungen: Mi 19:00 Uhr (im Netz, Zwangsabgabe nur, wenn auch korrigiert wird, wird vorher angekündigt)
- Abgabe: in der Regel keine
- Kleingruppenarbeit ist sinnvoll und wird eingefordert, 2 Personen je Gruppe
- **Sie können nur durch „Selbermachen“ lernen, nur lesen und zuhören reicht nicht!**

Maple

Properties

- Software package
- implemented in the programing language C
- available for many Operating Systems, e.g. Windows, Unix, Linux
- designed for numerical and **symbolic** expressions

- includes untilities for algebra, calculus, discrete mathematics, graphics, ...

History

- 1980: first development at the University of Waterloo, Canada
- 1988: Waterloo Maple Software was founded in order to sell and improve the software
- currently: version 14

Getting started

Under Windows, we find the program Maple in the start menu, under Mac OS the program can be found in the
application directory. Under Linux, we start Maple via the command xmaple inside a termional shell:

- login to one of the machines in K313 or K309 in the Math building
- open a shell / a terminal
- type: xmaple (or maple, if you would like to work without windows; e.g. remote from home)

Menu bar at the top:

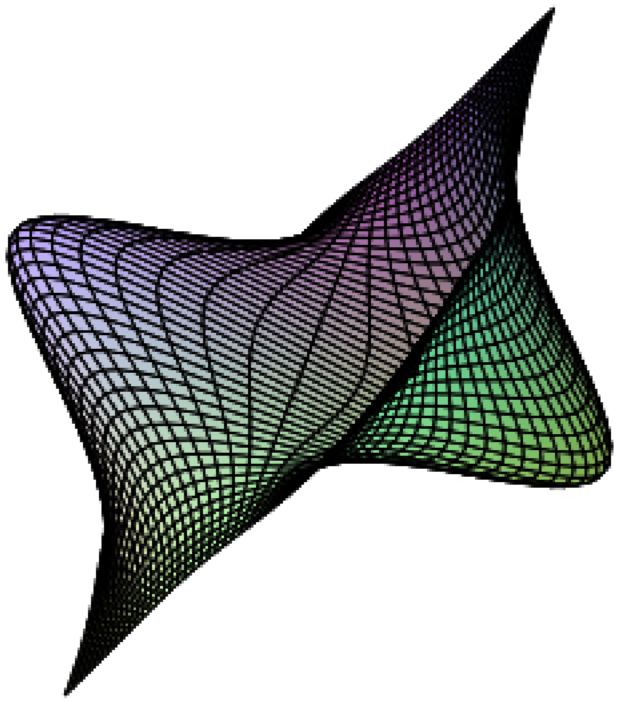
- allows you to save or load and edit your maple session
  e.g. clicking on the ***File*** menu and selecting ***Save*** allows to save the current worksheet
- below the menu bar, there is a collection of shortcut-buttons

Maple Help

- help menu, "Maple Help"
- ?command; e.g. ?solve, if you know the keyword in advance

- the help-window has two panels: the Help Navigator on the left and the help itself on the right
- each help page contains some examples; copying an example and pasting it into the worksheet is possible

$restart; plot3d\left(x\,\mathrm{e}^{-x^2-y^2}, x=-2..2, y=-2..2, grid=[49,49]\right);$



Basic Conventions

Entering a command, example

```
> restart;
> 3 + 4;
```

7 **(1)**

Arithmetic operators

Addition	+	$3+4$
Substraction	-	$x-y$

Multiplication	*	2*x
Division	/	x / y
Exponentiation	^	3^4
Factorial	!	3!

The precedence order follows the mathematical conventions:

> $(10-1)\cdot 2$

$$18 \quad (2)$$

> $10-1\cdot 2$

$$8 \quad (3)$$

>

Special commands to access previous results
% latest one
%% second most recent
%%% third most recent

> *#this is a comment*

> $2\cdot 4$; # *most recent result becomes 8*

$$8 \quad (4)$$

> *%* $\cdot$ 12.4; # *this computes 8·12.4. 99.2 becomes most recent result*

$$99.2 \quad (5)$$

> *%% − %*; # *computes 8-99.2*

$$-91.2 \quad (6)$$

> $\frac{13.0}{2}+\frac{23}{7}$;

$$9.785714286 \quad (7)$$

Floating point representation, Computing with Decimal Numbers

There are several possibilities how to enforce floating point representations. Most explicit is the command evalf.

> $\mathit{evalf}\left(\frac{13}{2}+\frac{27}{13}\right)$;

$$8.576923077 \quad (8)$$

There are hundreds of such functions / procedures / commands.

Moreover, there are pre-defined variables like Digits.

> $Digits := 20;$

$$Digits := 20 \quad \textbf{(9)}$$

> $evalf\left(\frac{13}{2}+\frac{27}{13}\right);\ \frac{1.0}{3};\ \frac{1000.0}{3};$

$$8.5769230769230769231$$
$$0.33333333333333333333$$
$$333.33333333333333333 \quad \textbf{(10)}$$

> $Digits := 10;$

$$Digits := 10 \quad \textbf{(11)}$$

> 10000!;

2846259680917054518906413212119868890148051401702799230794179994274411340003 7\ **(12)**
64443772990786757784775[...35460 digits...]
000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000

Expressions, functions and variables, Symbolic Computing

Defining Expressions with ":="
- expression: combination of numbers, variables and operators
- Syntax is *name:=expression*
- maybe most used concept in Maple

Example

> *restart*;

> $f := (1-x)^2 \cdot x^5;$

$$f := (1-x)^2\, x^5 \quad \textbf{(13)}$$

> $f;$

$$(1-x)^2\, x^5 \quad \textbf{(14)}$$

If you make a mistake, you can go back with the cursor, change the command-line and re-execute the line.

Be careful and check whether the program "understands" what you expect it to understand!

>

> $a := 3;$

$$a := 3 \quad \textbf{(15)}$$

> $b := a;$

$$b := 3 \quad \textbf{(16)}$$

> $a := a + 1;$

$$a := 5 \quad (17)$$

> $b;$

$$3 \quad (18)$$

> $a;$

$$5 \quad (19)$$

> $b := c;$

$$b := c \quad (20)$$

> $c := 3;$

$$c := 3 \quad (21)$$

> $b;$

$$3 \quad (22)$$

> $c := 5;$

$$c := 5 \quad (23)$$

> $b;$

$$5 \quad (24)$$

> $c := c + 1;$

$$c := 6 \quad (25)$$

> $b;$

$$6 \quad (26)$$

> $g := x \rightarrow x^2;$ # *a 'function'*

$$g := x \rightarrow x^2 \quad (27)$$

> $g(3);$

$$9 \quad (28)$$

> $h := (x, y) \rightarrow x^2 - 2 \cdot y^{\frac{1}{3}};$

$$h := (x, y) \rightarrow x^2 - 2\, y^{1/3} \quad (29)$$

> $evalf(h(4, 3));$

$$13.11550086 \quad (30)$$

> $x := 2; f; unassign('x'); f;$

$$x := 2$$
$$32$$
$$(1 - x)^2 x^5 \quad (31)$$

> $eval(f, x = 2);$

$$32 \quad (32)$$

> $ff := unapply(f, x);$

$$ff := x \rightarrow (1 - x)^2 x^5 \quad (33)$$

> $ff(2);$

$$32 \quad (34)$$

```
> restart;
>
```

Complex Numbers

- a complex number z is of the form a + bi, with i^2= -1 and a,b ∈ℝ. a = Re(z) is the real part of z and b=Im(z) is the imaginary part of z. An equivalent definition is via a two dimensional vector (a,b).
- two complex numbers are equal if and only if their real parts and their imaginary parts are equal

- Complex numbers are added, subtracted, multiplied, and divided by formally applying the associative, commutative and distributive laws of algebra, together with the equation i^2 = -1.
  - Addition : (a+bi) + (c+di) = (a+c) + (b+d)i [in vector notation: (a,b) + (c,d) = (a+c, b+d)]
  - Substraction : (a+bi) - (c+di) = (a-c) + (b-d)i
  - Multiplication: $(a+bi)\cdot(c+di) = (ac-bd) + (bc+ad)i$
  - Division : $\frac{a+bi}{c+di} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$, with c or d not equal to 0

- with the given definitions of addition, substraction, multiplication, division, and
  - the additive identity (zero-element) 0 + 0i,
  - the multiplicative identity (one-element) 1 + 0i,
  - the addidive inverse of a number a + bi: -a - bi, and
  - the multiplicative inverse of a + bi: $\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}$,

  the complex numbers ℂ are a *field* (dt: Körper)

Numeric complex computations

> $\frac{(3+3\cdot I)}{(2+6\cdot I)}$;

$$\frac{3}{5} - \frac{3}{10}\,\mathrm{I} \qquad \textbf{(35)}$$

> $\left(\frac{3}{3^2+5^2} + \frac{(-5)}{3^2+5^2}\cdot I\right)\cdot(3+5\cdot I)$;

$$1 \qquad \textbf{(36)}$$

Symbolic complex computations
Simplifying an expression

> *restart*; $x := 2$;

$$x := 2 \qquad \textbf{(37)}$$

> $\left(\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}\cdot I\right)\cdot(a+b\cdot I)$;

(38)

$$\left(\frac{a}{a^2+b^2}-\frac{I\,b}{a^2+b^2}\right)(a+I\,b) \quad \textbf{(38)}$$

>

> *simplify*(%);

$$1 \quad \textbf{(39)}$$

Another example for simplify:

> sqrt(a^2);

$$\sqrt{a^2} \quad \textbf{(40)}$$

> *simplify*(sqrt(a^2));

$$\operatorname{csgn}(a)\,a \quad \textbf{(41)}$$

> sqrt(a^2) assuming $a < 0$;

$$-a \quad \textbf{(42)}$$

> *simplify*(sqrt(a^2)) assuming a :: *real*, $a > 0$;

$$a \quad \textbf{(43)}$$

> *simplify*(sqrt(a^2)) assuming a :: *real*;

$$|a| \quad \textbf{(44)}$$

>

The following expression leads to a surprising answer. Why? Because somewhere above, we already defined x. Thus: be careful and alert!

> *simplify*($\sin(x)^2 \cdot x^4 + \cos(x)^2 \cdot x^4$);

$$16 \quad \textbf{(45)}$$

> *simplify*($\sin(y)^2 \cdot y^4 + \cos(y)^2 \cdot y^4$);

$$y^4 \quad \textbf{(46)}$$

> *restart*;

> *simplify*($\sin(x)^2 \cdot x^4 + \cos(x)^2 \cdot x^4$);

$$x^4 \quad \textbf{(47)}$$