Problem 1: plot and plot3d

a)

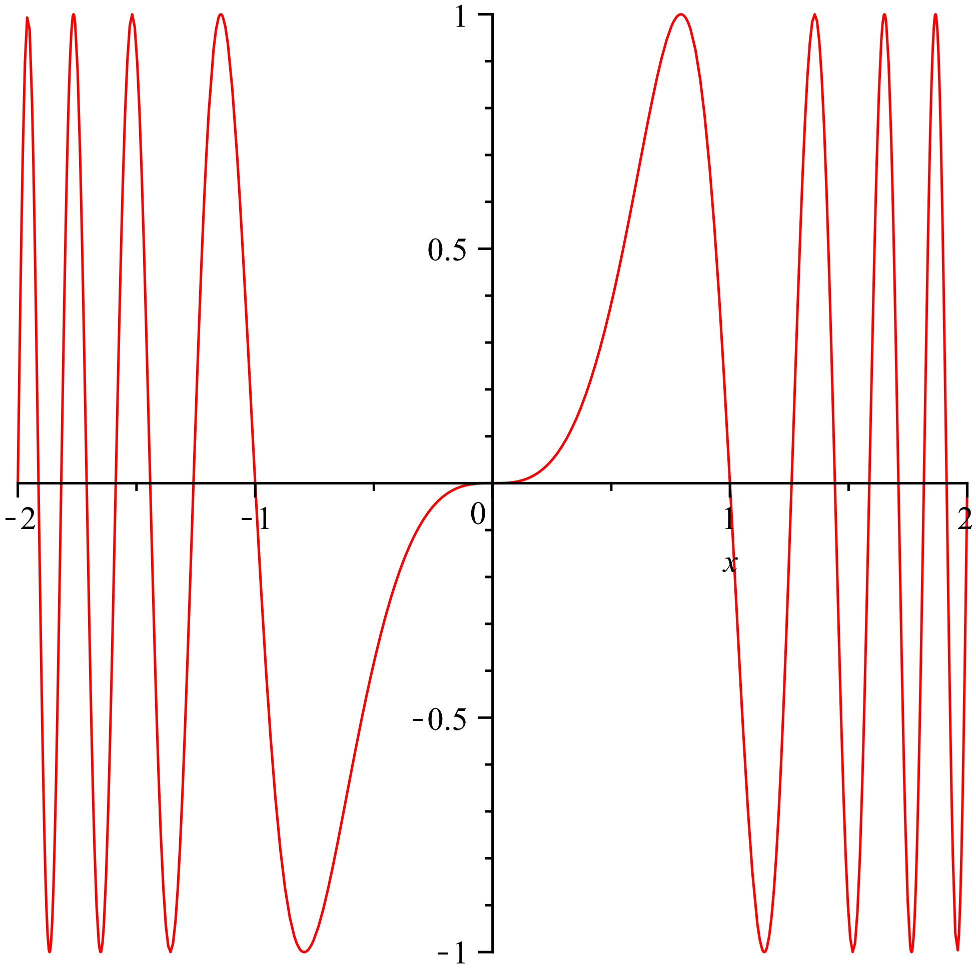
$f := x \rightarrow \sin(\pi \cdot x^3)$

$$x \rightarrow \sin(\pi x^3) \quad (1.1.1)$$

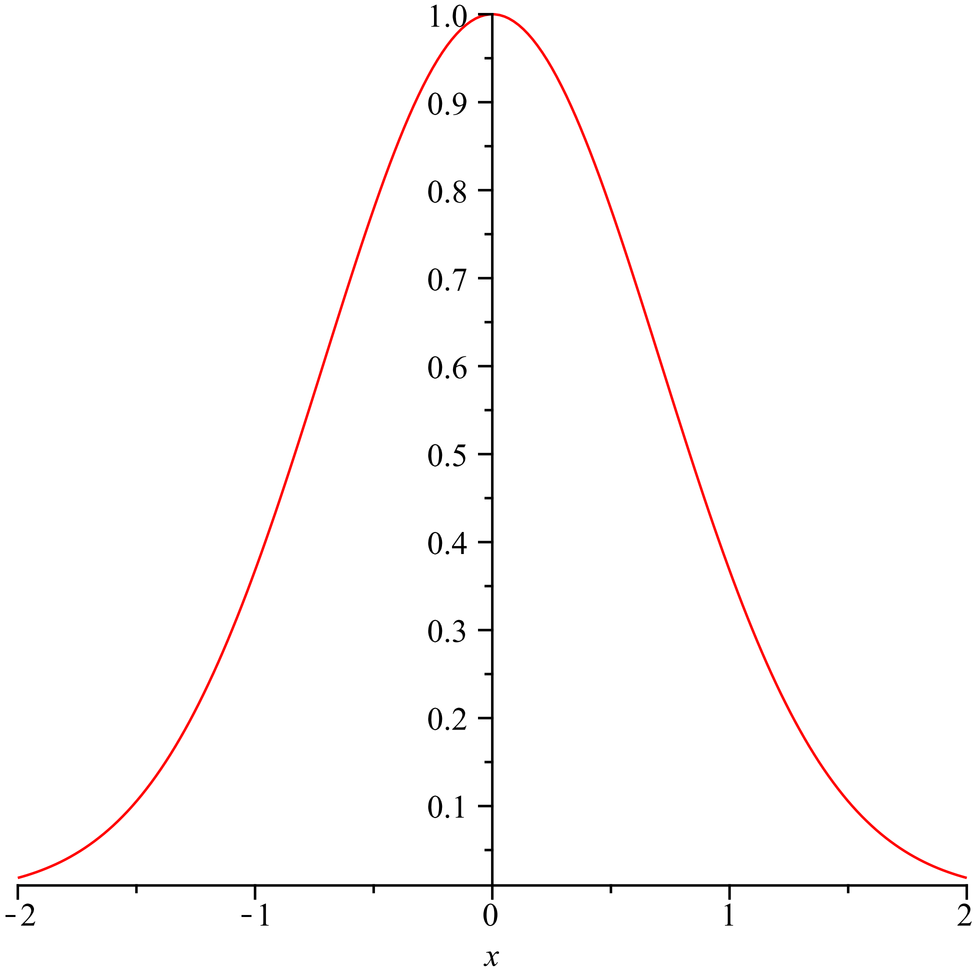
$g := x \rightarrow e^{-x^2}$

$$x \rightarrow e^{-x^2} \quad (1.1.2)$$

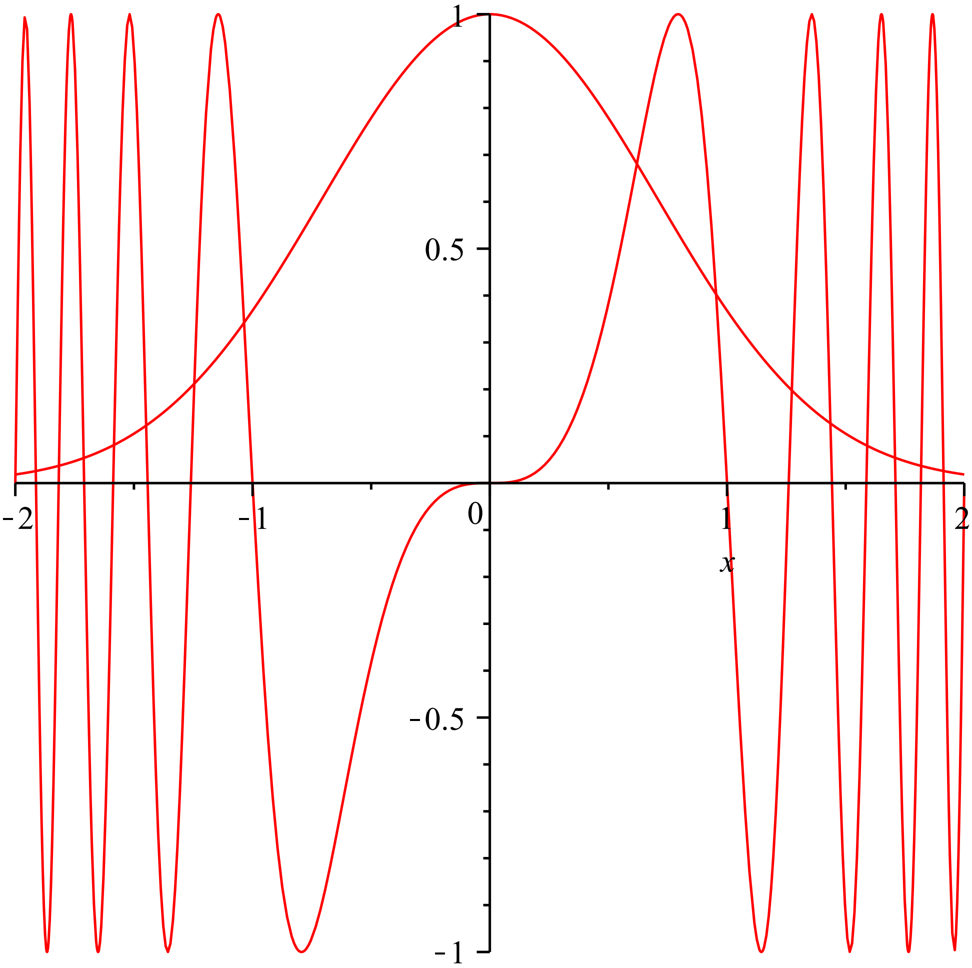
$P1 := plot(f(x), x = -2..2) : P1;$



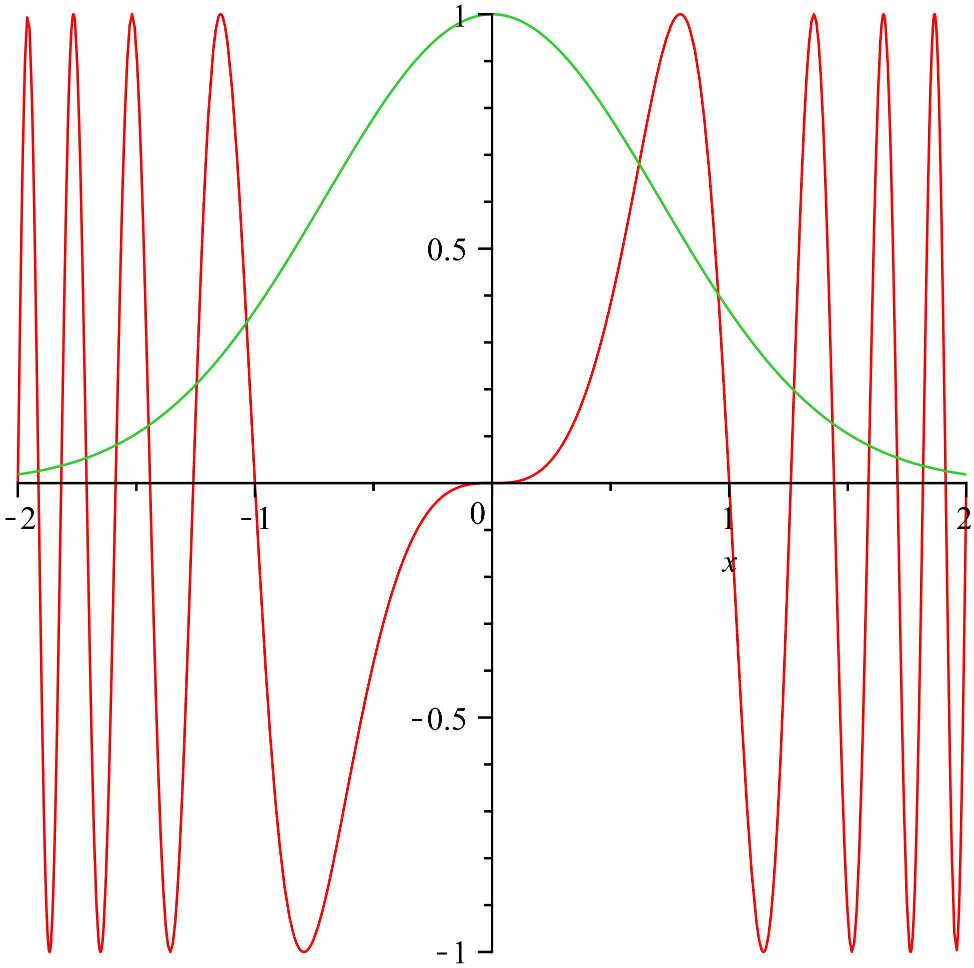
$P2 := plot(g(x), x = -2..2) : P2;$



$with(plots):$
$display(\{P1, P2\})$



$plot([f(x), g(x)], x=-2..2)$

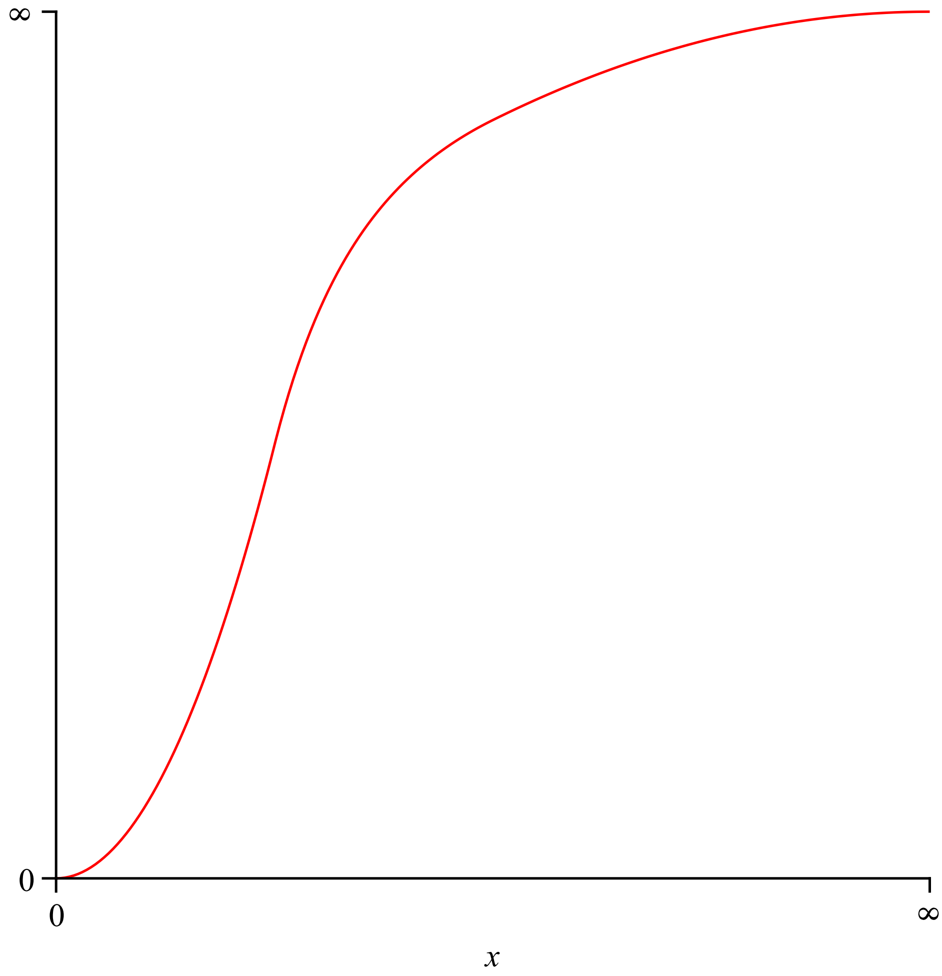


b)

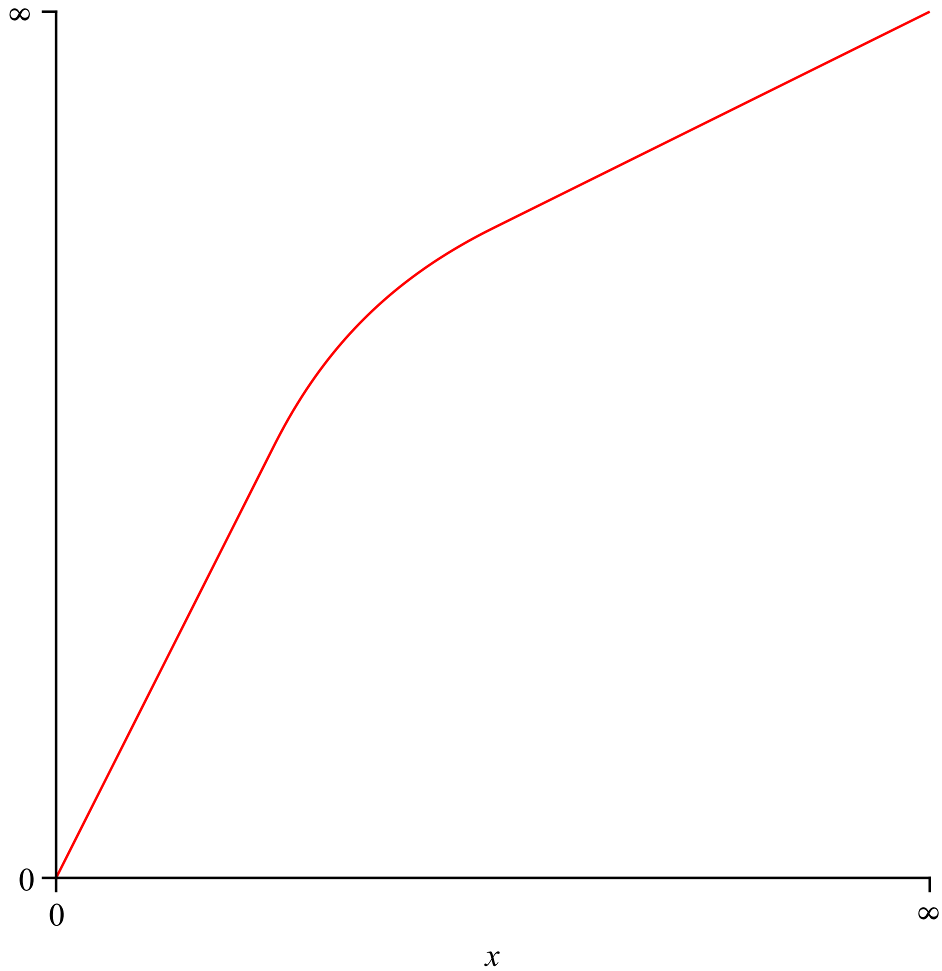
$h := x \rightarrow x^2$

$$x \rightarrow x^2 \qquad \textbf{(1.2.1)}$$

$plot(h(x), x = 0 .. \infty)$



$plot(h'(x), x = 0 .. \infty)$

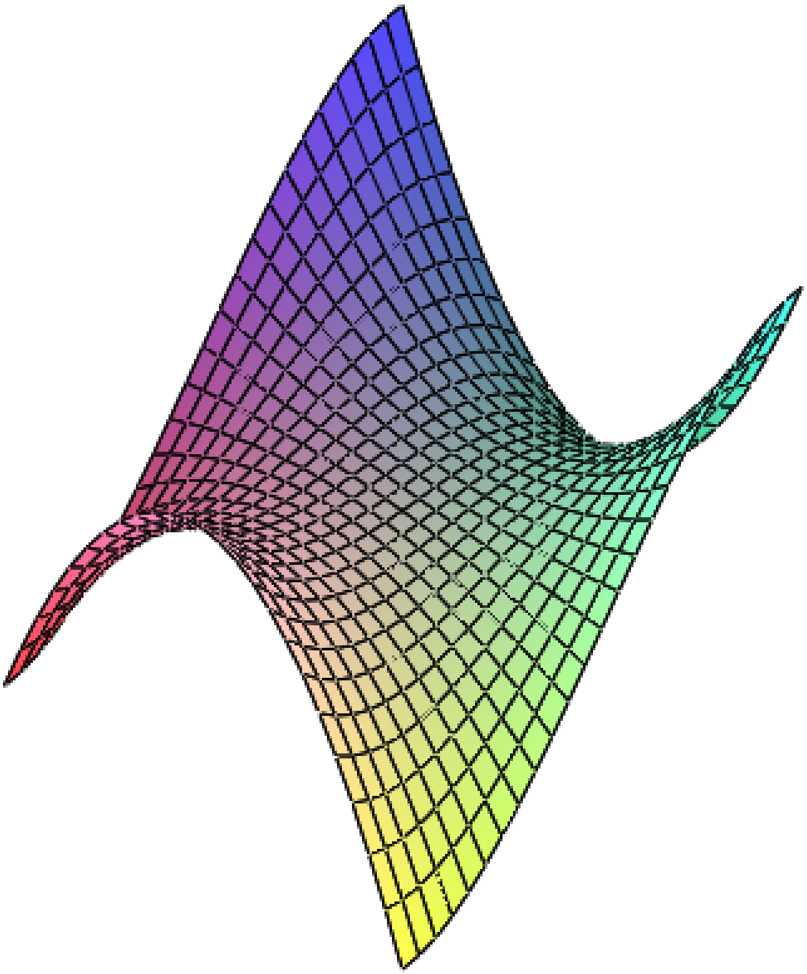


c)

$MonkeySaddle := (x, y) \rightarrow x^3 - 3 \cdot x \cdot y^2$

$$(x, y) \rightarrow x^3 - 3\, x\, y^2 \qquad \textbf{(1.3.1)}$$

$plot3d(MonkeySaddle(x, y), x = -2 \,..\, 2, y = -2 \,..\, 2)$



Problem 2: Lists and Sets in Maple

a)

A list is an ordered sequence of expressions enclosed in square brackets [ ]. The ordering of the expressions is the ordering of the sequence.
A set is an unordered sequence of distinct expressions enclosed in braces { }, representing a set in the mathematical sense.
Sets have a deterministic ordering that, for most objects, is not based on runtime properties. This means that when {b,c,a} is input, the order will be fixed to {a,b,c} no matter when you created that set. A notable exception to this rule is when a set contains multiple mutable objects of the same type. For example, two vectors inside a set could appear in either order in different sessions.

b)

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with(numtheory) :
divisors(23545800) intersect divisors(25491186) intersect divisors(229420674)
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$$\{1, 2, 3, 6, 9, 18, 127, 254, 381, 762, 1143, 2286\} \quad (2.2.1)$$

divisors(*igcd*(23545800, 25491186, 229420674))

$$\{1, 2, 3, 6, 9, 18, 127, 254, 381, 762, 1143, 2286\} \quad (2.2.2)$$

c)

$$M := \left[solve\left(x^4 - 4\cdot x^3\cdot\pi + \frac{26}{9}\cdot x^2\cdot\pi^2 + \frac{4}{9}\cdot x\cdot\pi^3 - \frac{1}{3}\cdot\pi^4\right)\right]$$

$$\left[\pi, 3\,\pi, \frac{1}{3}\,\pi, -\frac{1}{3}\,\pi\right] \quad (2.3.1)$$

i)

map(sin, *M*)

$$\left[0, 0, \frac{1}{2}\sqrt{3}, -\frac{1}{2}\sqrt{3}\right] \quad (2.3.1.1)$$

ii)

sin~(*M*)

$$\left[0, 0, \frac{1}{2}\sqrt{3}, -\frac{1}{2}\sqrt{3}\right] \quad (2.3.2.1)$$

Problem 3: Solving Systems of Linear Equations

$G1 := 2\cdot x + 8\cdot y + 4\cdot z = 7$

$$2\,x + 8\,y + 4\,z = 7 \quad (3.1)$$

$G2 := 6\cdot x + 2\cdot y + 4\cdot z = 9$

$$6\,x + 2\,y + 4\,z = 9 \quad (3.2)$$

$G3 := x + z = 8$

$$x + z = 8 \quad (3.3)$$

$G4 := 3\cdot x + 8\cdot y + 5\cdot z = 15$

$$3\,x + 8\,y + 5\,z = 15 \quad (3.4)$$

$G5 := 3\cdot x + 8\cdot y + 5\cdot z = 9$

$$3\,x + 8\,y + 5\,z = 9 \quad (3.5)$$

solve({*G1*, *G2*, *G3*})

$$\left\{x = -\frac{67}{10}, y = -\frac{24}{5}, z = \frac{147}{10}\right\} \quad (3.6)$$

solve({*G1*, *G2*})

$$\left\{x = \frac{1}{2} + \frac{3}{2}\,y, y = y, z = -\frac{11}{4}\,y + \frac{3}{2}\right\} \quad (3.7)$$

solve({*G1*, *G2*, *G4*, *G3*})

$$\left\{x = -\frac{67}{10}, y = -\frac{24}{5}, z = \frac{147}{10}\right\} \quad (3.8)$$

solve({*G1*, *G2*, *G5*, *G3*})
⇒The last system does not have any solution!

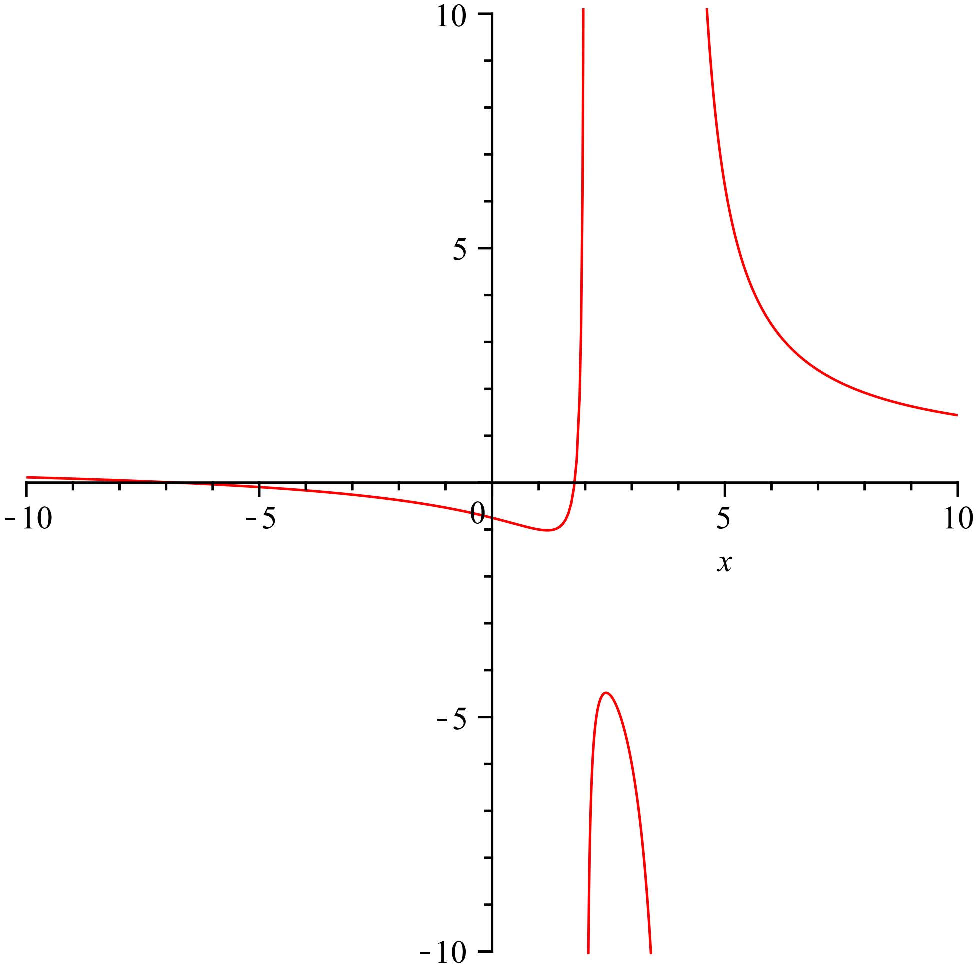
Problem 4: Curve Sketching

$f := x \to \frac{x^2 + 5 \cdot x - 12}{2 \cdot x^2 - 12 \cdot x + 16}$

$$x \to \frac{x^2 + 5\,x - 12}{2\,x^2 - 12\,x + 16} \tag{4.1}$$

a)

plot(*f*(*x*), *view* = [-10 ..10, -10 ..10], *discont*)



b)

$solve(2 \cdot x^2 - 12 \cdot x + 16 \neq 0)$

$$\{x \neq 2, x \neq 4\} \quad \textbf{(4.2.1)}$$

c)

solve(*f*(*x*))

$$-\frac{5}{2} + \frac{1}{2}\sqrt{73}, -\frac{5}{2} - \frac{1}{2}\sqrt{73} \quad \textbf{(4.3.1)}$$

d)

$E := [solve(f'(x))]$

$$\left[\frac{20}{11} - \frac{4}{11}\sqrt{3}, \frac{20}{11} + \frac{4}{11}\sqrt{3}\right] \quad \textbf{(4.4.1)}$$

$evalf(f''(E[1]))$

$$1.330313028 \quad \textbf{(4.4.2)}$$

⇒Minimum:

$evalf([E[1], f(E[1])])$

$$[1.188345161, -1.017949192] \quad \textbf{(4.4.3)}$$

$evalf(f''(E[2]))$

$$-14.33031280 \quad \textbf{(4.4.4)}$$

⇒Maximum:

$evalf([E[2], f(E[2])])$

$$[2.448018475, -4.482050791] \quad \textbf{(4.4.5)}$$

e)

$W := [solve(f''(x))]$

$$\left[-\frac{4}{11}18^{1/3} - \frac{2}{33}18^{2/3} + \frac{20}{11}, \frac{2}{11}18^{1/3} + \frac{1}{33}18^{2/3} + \frac{20}{11} + I\sqrt{3}\left(-\frac{2}{11}18^{1/3} + \frac{1}{33}18^{2/3}\right), \frac{2}{11}18^{1/3} + \frac{1}{33}18^{2/3} + \frac{20}{11} - I\sqrt{3}\left(-\frac{2}{11}18^{1/3} + \frac{1}{33}18^{2/3}\right)\right] \quad \textbf{(4.5.1)}$$

$wp := W[1]$

$$-\frac{4}{11}18^{1/3} - \frac{2}{33}18^{2/3} + \frac{20}{11} \quad \textbf{(4.5.2)}$$

Another interesting solution:

use *RealDomain* **in** $wp := solve(f''(x))$ **end use**

$$-\frac{4}{11}18^{1/3} - \frac{2}{33}18^{2/3} + \frac{20}{11} \quad \textbf{(4.5.1.1)}$$

$evalf(f'''(wp))$

$$0.291917742 \quad (4.5.3)$$

⇒Inflection point:

evalf([*wp*, *f*(*wp*)])

$$[0.448925223, -0.8672721812] \quad (4.5.4)$$

f)

$$\left| \int_{-\frac{5}{2} - \frac{1}{2}\sqrt{73}}^{-\frac{5}{2} + \frac{1}{2}\sqrt{73}} f(x)\, dx \right|$$

$$-\frac{1}{2}\sqrt{73} + 6\ln\left(13+\sqrt{73}\right) - \frac{1}{2}\ln\left(9+\sqrt{73}\right) - 6\ln\left(13-\sqrt{73}\right) + \frac{1}{2}\ln\left(9 - \sqrt{73}\right) \quad (4.6.1)$$

evalf(%)

$$3.358087877 \quad (4.6.2)$$