Introduction to Mathematical Software Course Attendance Certificate

TECHNISCHE UNIVERSITÄT DARMSTADT

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Week: 07.02.2011 - 11.02.2011

The Rules

This is the special exercise sheet which you have to work on for receiving the attendance certificate for this course.

- The assignment is due in the exercise sessions of the week 07.02.2011 - 11.02.2011 .

- You are supposed to work in the groups that you have formed for the exercise classes. However, we will examine at most 3 students together, therefore if you have a larger group, you will have to split up.

- Send an email to lorenz@mathematik.tu-darmstadt.de with the subject **[IMS2010]**, containing your immatriculation numbers (one number per line) and an attachment with your Maple worksheet.

  The submission deadline is 24 hours before the beginning of the exercise class in which you are tested! Note that we can only test you if we have received your email beforehand!

- There will be only the marks "passed" and "not passed".

- In order to pass, it is necessary to solve all exercises!

- In the exercise sessions, the results will be discussed group by group. Only those members of a group can "pass" who actively participate in this short discussion. Every group member should be able to explain any part of the exercise.

- **Visit the course homepage regularly in case there are corrections or additional hints for the exercises!**

The Regular Icosahedron

A regular icosahedron is a convex regular polytope with 20 triangular faces, 30 edges and 12 vertices. Five triangular faces meet at each of the twelve vertices.
Let $v_1, \ldots, v_{12}$ denote the vertices

$$\left\{ \begin{array}{llll} v_1 = (0, \tau, 1), & v_2 = (0, \tau, -1), & v_3 = (0, -\tau, 1), & v_4 = (0, -\tau, -1), \\ v_5 = (1, 0, \tau), & v_6 = (1, 0, -\tau), & v_7 = (-1, 0, \tau), & v_8 = (-1, 0, -\tau), \\ v_9 = (\tau, 1, 0), & v_{10} = (\tau, -1, 0), & v_{11} = (-\tau, 1, 0), & v_{12} = (-\tau, -1, 0) \end{array} \right\}, \tag{1}$$

where τ is the *golden ratio* $\tau = (1 + \sqrt{5})/2$.
In general, a convex hull of n points, denoted as $\mathrm{conv}(v_1, \ldots, v_n)$, is defined as

$$\left\{ \sum_{i=1}^{n} \lambda_i v_i \;\middle|\; \lambda_i \geq 0, \sum \lambda_i = 1 \text{ with } 1 \leq i \leq n \right\}.$$

The convex hull of $v_1, \ldots, v_{12}$ defines an icosahedron, which we call I. If you unfold the polytope, it will look like shown in Figure 1.
For additional information, see http://mathworld.wolfram.com/Icosahedron.html and
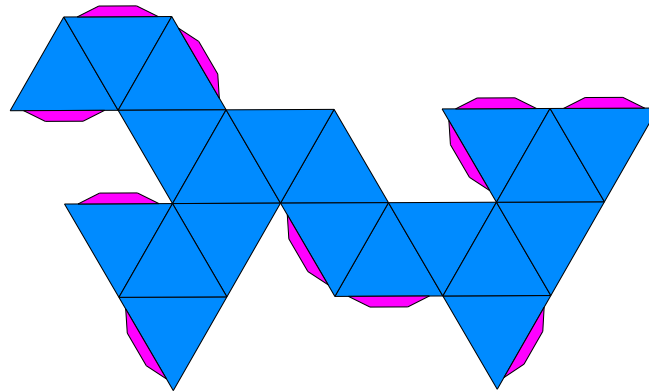http://www.mathematische-basteleien.de/ikosaeder.htm (in German).

Figure 1: An unfolded icosahedron (with flaps such that you can glue it).

The Exercises

Problem 1 Drawing and Volume of the Regular Icosahedron

Draw the icosahedron I and determine its exact volume.

Problem 2 Perturbed Icosahedra

Write a Maple-program that computes the volume of a *perturbed icosahedron*. This means that another person gives you perturbations $e_1, \ldots, e_{12}$ where $e_i \in B_\varepsilon(0)$, $\varepsilon \geq 0$, of the 12 vertices with

$$B_\varepsilon(0) := \{x \in \mathbb{R}^3 \,|\, \|x\| \leq \varepsilon\},$$

and your program is supposed to compute the volume of the perturbed icosahedron given by

$$\mathrm{conv}(v'_1 = v_1 + e_1, \ldots, v'_n = v_n + e_n),$$

with $v_1, \ldots, v_{12}$ given by (1). Here, $\|\cdot\|$ denotes the Euclidean norm. Note that, for small values of ε, the convex hull of the vertices will still be an icosahedron.

Remarks:

- Stick to the order of the vertices as given in (1).

- Your problem has to be correct for all $\varepsilon \leq 0.23$ (We will test your program for several values)

- To test your program, the volume of the perturbed icosahedron with $\{e_1, e_2, \ldots e_{12}\}$ given by

$$\left\{ \begin{pmatrix} 0 \\ 0.1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.03 \\ 0.1 \\ -0.07 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -0.1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.13 \\ 0.07 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0.23 \end{pmatrix}, \begin{pmatrix} -0.17 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -0.1 \\ 0.1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.1 \\ -0.03 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

is approximately 18.07014098.

Problem 3 Documentation

In your Maple worksheet, document your program **in English**.

A man is in a hot air balloon and realizes that he has lost his way. He reduces height. When he discovers another man on the ground he calls out to him: «Excuse me, can you help me? I promised a friend to meet him half an hour ago, but I don't know where I am.»

The man on the ground thinks for a while and says then: «You're in a hot air balloon.»

«You must be a mathematician.»

«I am, how do you know?»

«You see, you needed forever to get a result. You answer is completely correct. And it is completely useless.»

The mathematician replies: «You must be a manager.»

«Oh yes, I am. How did you know?»

«You see, you don't know where you are and where you are going. You have promised something, you have no idea how to keep this promise, and you expect me to solve your problem. As a matter of fact you are in completely the same position as before meeting me, but somehow it is my fault now.»