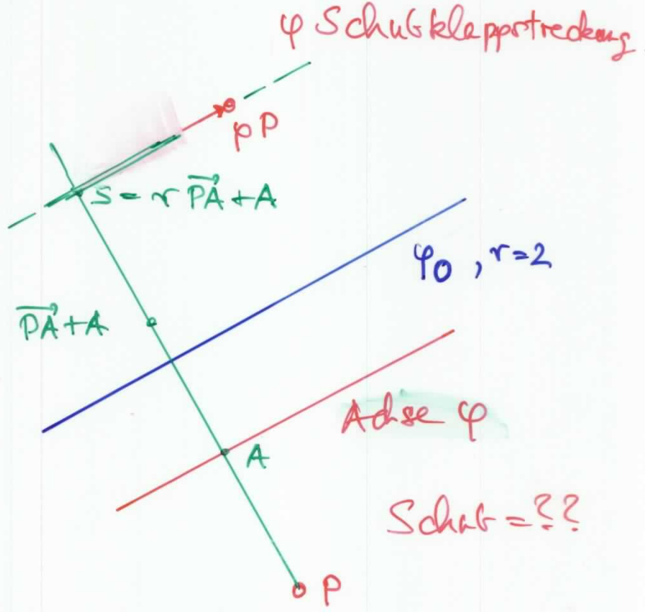
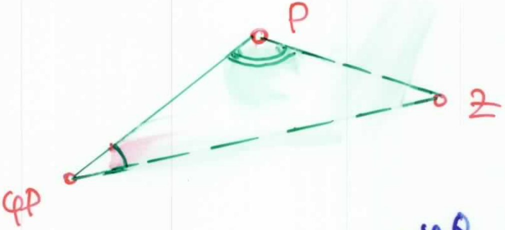
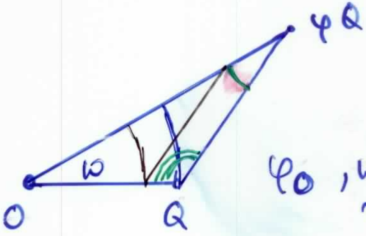
14.8



$$S = r\overrightarrow{PA} + A = r\overrightarrow{PA} + \overrightarrow{PA} + P$$
$$= (r+1)\overrightarrow{PA} + P$$
$$\overrightarrow{PA} = \frac{1}{r+1}\overrightarrow{PS}$$

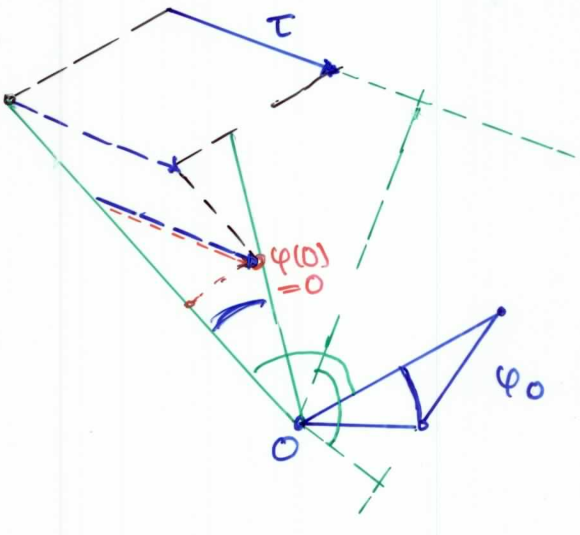
φ Drehstreckung





φ_O, $\omega = 30°$
$r = 2$

$$\tau \circ \varphi_0 = \varphi$$



quae centrum habeat in puncto I, quo facilius istam investigationem ad doctrinam sphaericam traducere liceat.

THEOREMA

Quomodocunque sphaera circa centrum suum convertatur, semper assignari potest diameter, cuius directio in situ translato conveniat cum situ initiali.

DEMONSTRATIO

25. Referat (Fig. 2) circulus A, B, C circulum sphaerae maximum quemcunque, in statu initiali, qui facta translatione pervenerit in situm a, b, c, ita ut puncta A, B, C translata sint in puncta a, b, c; punctum autem A sit simul intersectio horum duorum circulorum. Quo posito demonstrandum est semper dari punctum O, quod pari modo referetur tam ad circulum A, B, C quam ad circulum a, b, c. Ad hoc igitur necesse est, ut primo distantiae OA et Oa sint inter se aequales; deinde vero, ut etiam arcus OA et Oa ad illos duos circulos aequaliter sint inclinati, sive ut sit angulus $Oab =$ angulo OAB: erunt ergo etiam complementa ad duos rectos, hoc est anguli OaA et $OA\alpha$ inter se aequales. Quoniam autem arcus Oa et OA sunt aequales, erit quoque angulus $OaA =$ angulo OAa, ideoque $OAa = OA\alpha$; unde patet, si angulus $aA\alpha$ bisecetur arcu OA,

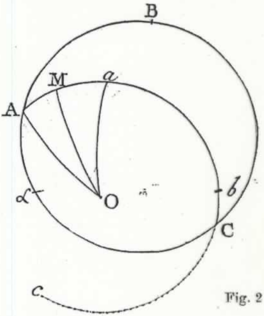


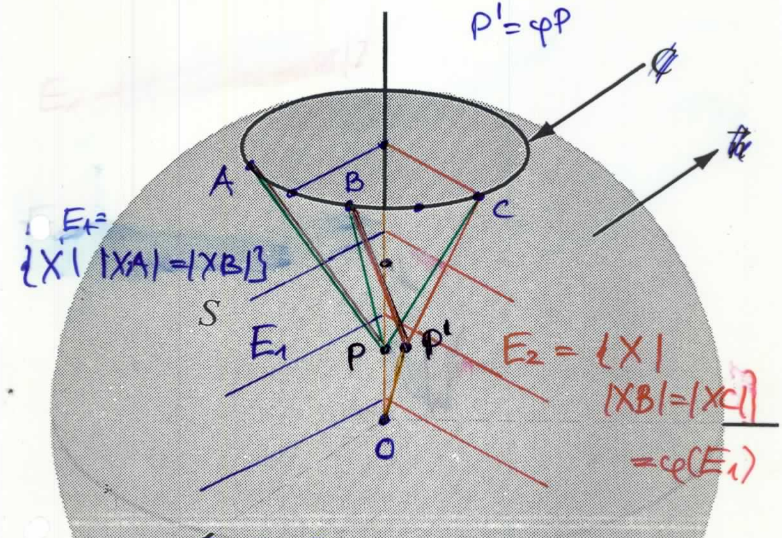
Fig. 2

tum punctum quaesitum O alicubi in isto arcu AO fore situm; quod igitur reperietur si arcus aO ita ducatur, ut angulus AaO aequalis evadat angulo OAa. Intersectio enim horum arcuum dabit punctum O, per quod si ducatur diameter Sphaerae, eius positio in situ translato etiamnunc eadem erit, quae fuerat in situ initiali.

26. Ad hoc punctum O facilius definiendum bisecari potest arcus Aa in puncto M, ubi constituatur arcus MO ad Aa normalis; tum vero ducatur arcus AO, ita ut angulum $aA\alpha$ bisecet; atque intersectio horum arcuum O monstrabit punctum quaesitum. Hic observatur, si arcus $a\alpha$ aequalis capiatur arcui aA, fore α punctum Sphaerae, quod facta translatione pervenerit in punctum A,

$B = \varphi A \quad C = \varphi B$

$P' = \varphi P$



$P \in E_1 \cap E_2$

$|PA| = |PB| = |PC|$

$|P'B| = |P'C| \quad (P \in E_1 \Rightarrow P' \in E_2)$

$|P'B| = |\varphi P \, \varphi A| = |PA|$

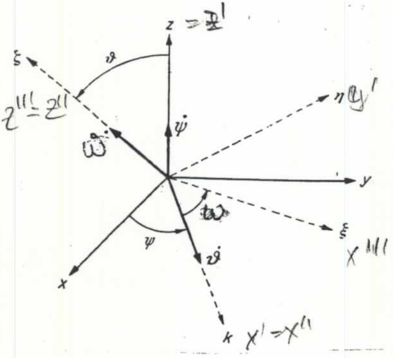
$|PO| = |P'O| \qquad OBCP \equiv OBCP'$

$\Longrightarrow P' = P$ wegen Orientierung

$$\varphi = \varphi_3 \circ \varphi_2 \circ \varphi_1$$

$$\begin{pmatrix} \cos\omega\cos\psi - \sin\omega\sin\psi\cos\theta & -\sin\omega\cos\psi - \cos\omega\sin\psi\cos\theta & \sin\psi\sin\theta \\ \cos\omega\sin\psi + \sin\omega\cos\psi\cos\theta & -\sin\omega\sin\psi + \cos\omega\cos\psi\cos\theta & -\cos\psi\sin\theta \\ \sin\omega\sin\theta & \cos\omega\sin\theta & \cos\theta \end{pmatrix}$$

$$\varphi_1 : \ \omega, \vec{e}_3$$
$$\varphi_2 : \ \theta, \vec{e}_1$$
$$\varphi_3 : \ \psi, \vec{e}_3$$



Dieselbe Drehung (mit derselben Matrix) kann man aber auch als Hintereinanderausführung $\chi_3 \circ \chi_2 \circ \chi_1$ von 3 Drehungen um zwei der (mitbewegten) Achsen a_1, a_3 eines Körpers sehen:

- a_3 liegt in Richtung $\vec{e}_3$ und χ_1 ist die Drehung um a_3 mit Winkel ψ
- a_1 liegt nun in Richtung von $\chi_1(\vec{e}_1)$ und χ_2 ist die Drehung um a_1 mit Winkel θ
- a_3 liegt nun in Richtung von $\chi_2\chi_1(\vec{e}_3)$ und χ_3 ist die Drehung um a_3 mit Winkel ω

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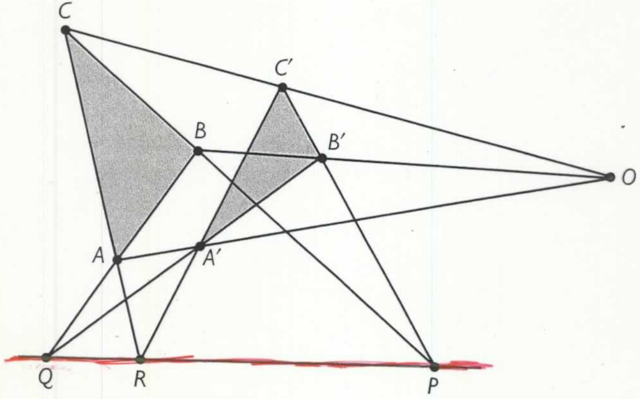

⇒ PQR collinear

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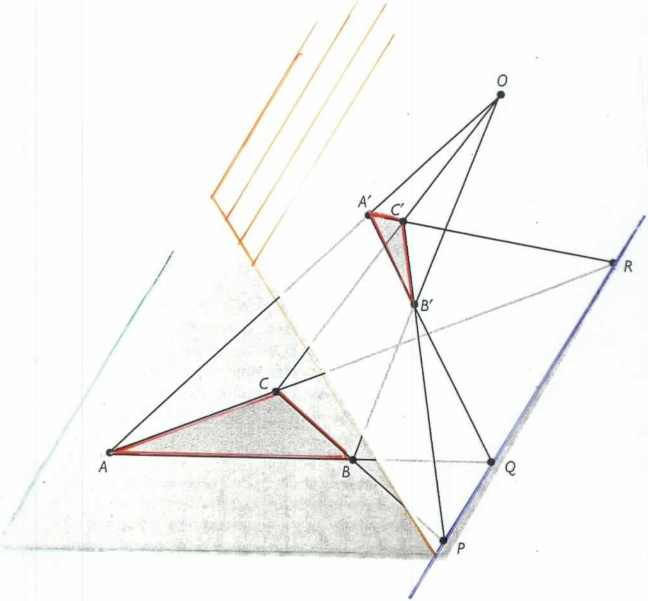

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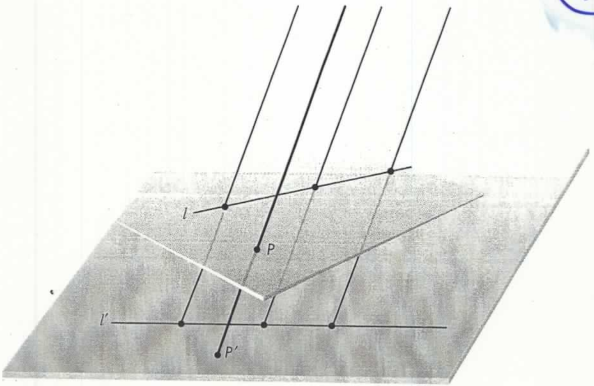

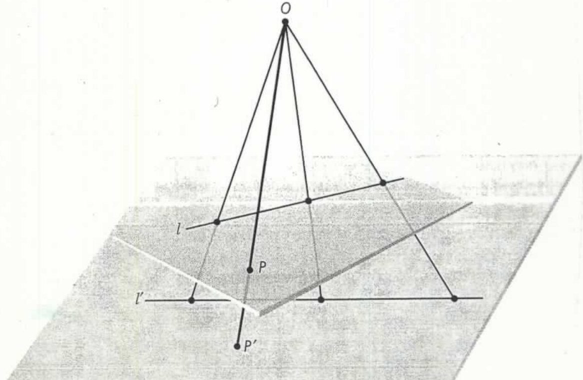

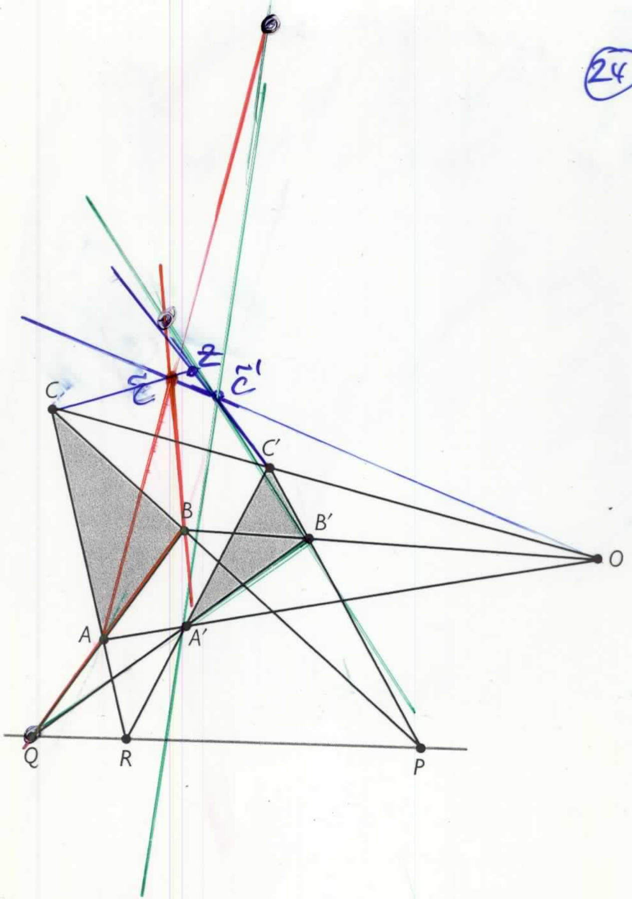
C
C′
B
B′
O
A
A′
Q
R
P