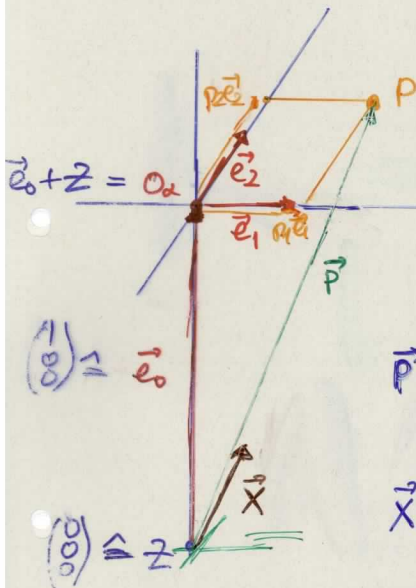


12.4 Homogene Koordinaten ^①

$$\vec{a} = \vec{e}_0, \vec{e}_1, \vec{e}_2$$

$$\vec{a} = \vec{e}_1, \vec{e}_2$$

$$p^\alpha = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$



$$\vec{p}^{\vec{a}} = \begin{pmatrix} 1 \\ p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 1 \\ p^\alpha \end{pmatrix}$$

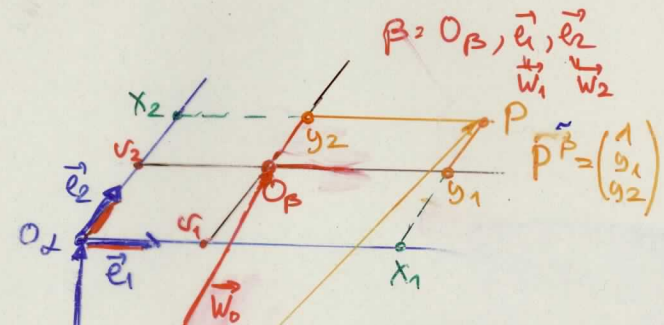
$$\vec{X} = \frac{1}{x_0} \vec{p} \sim \vec{p}$$

$$p_i = x_i / x_0 \quad i \geq 1$$

$$\vec{X} = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} \cdot x_0 \neq 0 \rightarrow p^{\vec{a}}$$

... Multiplikation mit $v \neq 0$

12.6 Transformation



$\beta = O_\beta, \vec{e}_1, \vec{e}_2$
 \vec{w}_1, \vec{w}_2

$$P_\beta = \begin{pmatrix} 1 \\ y_1 \\ y_2 \end{pmatrix}$$

$$P^\alpha = \begin{pmatrix} 1 \\ x_1 \\ x_2 \end{pmatrix}$$

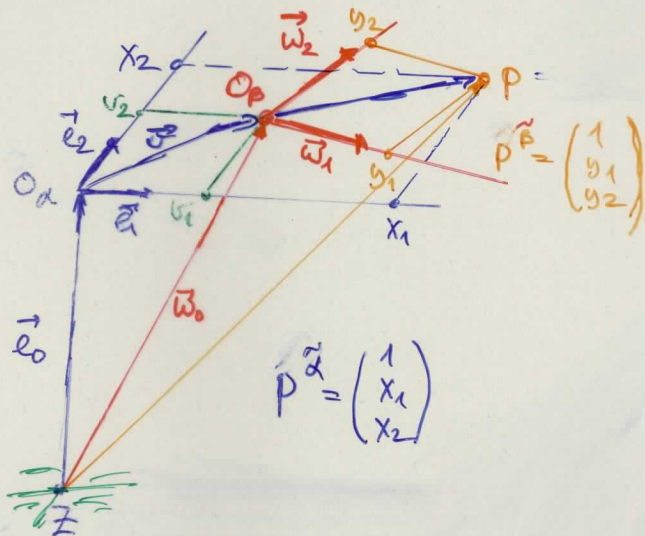
$$= \begin{pmatrix} 1 \\ v_1 + y_1 \\ v_2 + y_2 \end{pmatrix} = \tilde{T}_\beta \begin{pmatrix} 1 \\ y_1 \\ y_2 \end{pmatrix}$$

$$\tilde{T}_\beta = \begin{pmatrix} 1 & 0 & 0 \\ v_1 & 1 & 0 \\ v_2 & 0 & 1 \end{pmatrix}$$

W
 W

12.6. Transformation

(3)



$$P^{\tilde{\alpha}} = \begin{pmatrix} 1 \\ x_1 \\ x_2 \end{pmatrix}$$

$$P^{\tilde{\beta}} = \begin{pmatrix} 1 \\ y_1 \\ y_2 \end{pmatrix}$$

$$P^{\tilde{\alpha}} = \begin{pmatrix} 1 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ v_1 & \alpha^T \beta & \\ v_2 & & \end{pmatrix} \begin{pmatrix} 1 \\ y_1 \\ y_2 \end{pmatrix} = P^{\tilde{\beta}}$$

$\alpha^T \beta = \alpha^T \tilde{\beta}$

13. Affine Abbildungen

(7)

$$\vec{10E+Q}$$

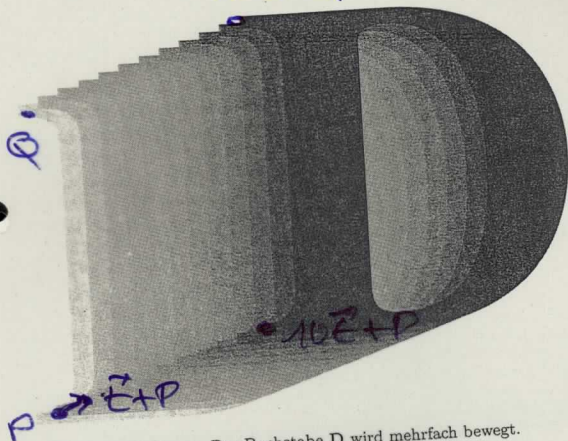


Abb. 6.3. Translationen: Der Buchstabe D wird mehrfach bewegt.

$$\tau_{\vec{t}}(P) = \vec{t} + P$$

Farin Hensford
Geometrie. Einführung in die Alg.

8

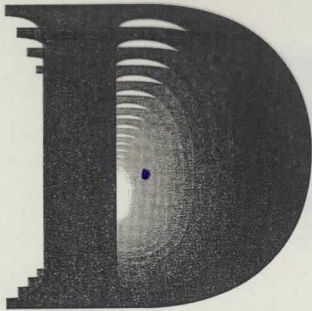


Abb. 6.4. Skalierung: der Buchstabe D wird mehrfach skaliert; der Ursprung liegt im Mittelpunkt.

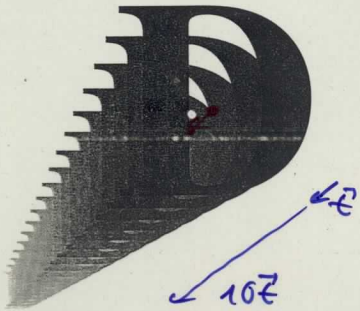


Abb. 6.5. Skalierung: der Buchstabe D wird mehrfach skaliert; nach jedem Schritt wurde zusätzlich eine Translation angewendet.

Farin Hansford Semestr.
Einf. Lh. Alg

5

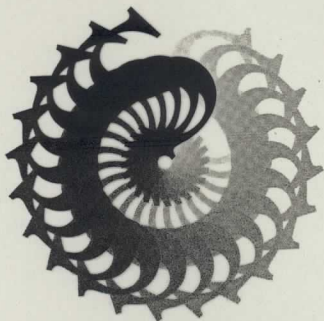


Abb. 6.6. Rotationen: Der Buchstabe S wird mehrfach rotiert; der Ursprung liegt unten links am Buchstaben.

6.5 Hintereinanderausführung affiner Abbildungen 117

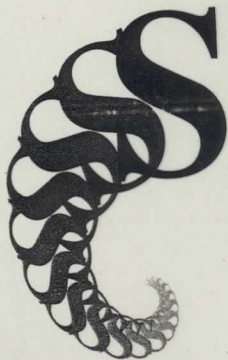
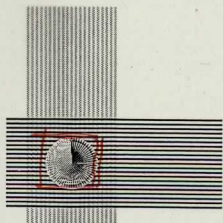
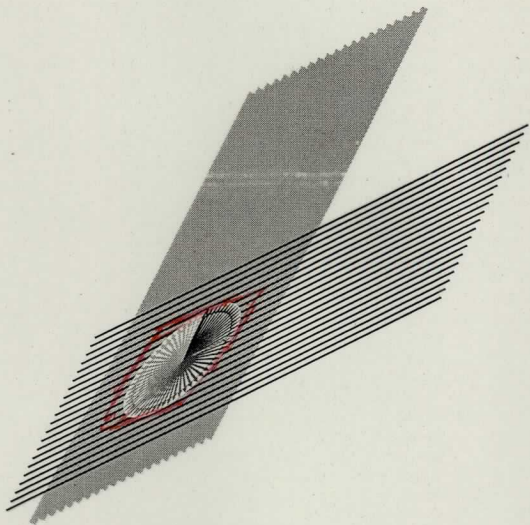


Abb. 6.7. Rotationen: Der Buchstabe S wird mehrfach rotiert; Skalierungen und Translationen werden ebenfalls angewendet.

10



M

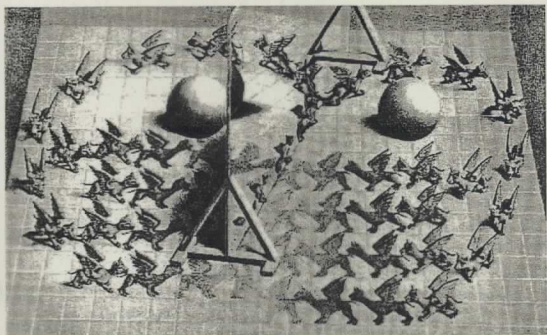


Abb. 6.8. M.C. Escher: Der magische Spiegel (1949).

118 6 Dinge in Bewegung setzen: Affine Abbildungen

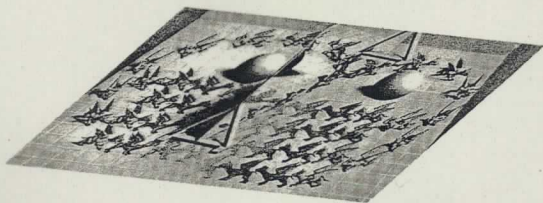
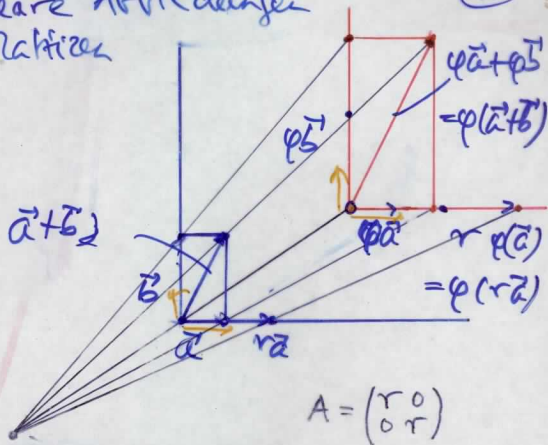


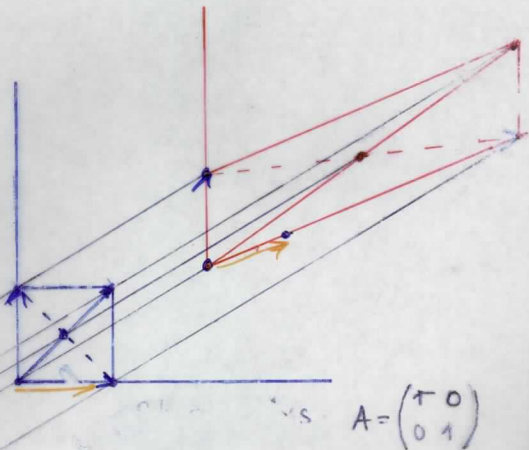
Abb. 6.9. Anwendung einer affinen Abbildung auf „den magischen Spiegel“.

133 Lineare Abbildungen
und Matrizen

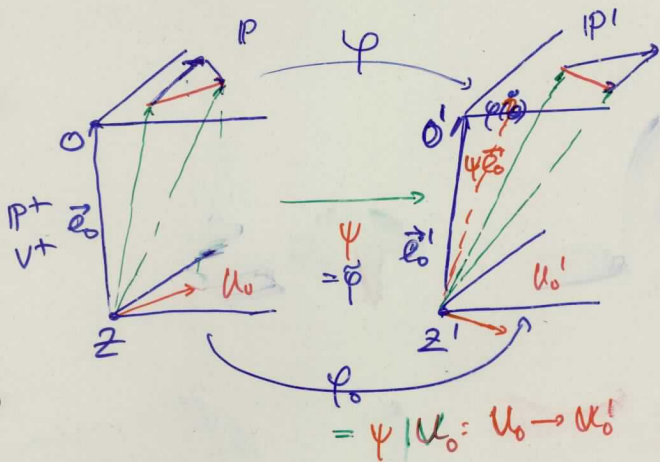
(13)



mit



13.4 Matrixbeschreibung (16)



$\varphi: P \rightarrow P'$ affin $\varphi(P) = \varphi(\vec{zP}) + z'$

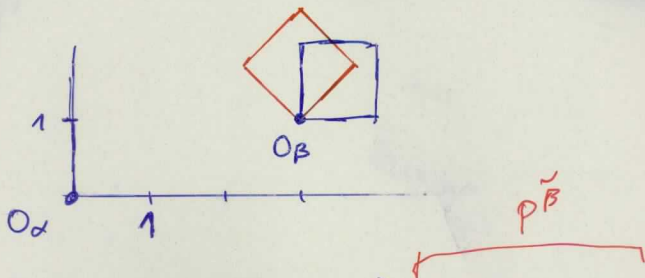
$\varphi: V \rightarrow V'$ linear $\psi|_{U_0} = \varphi_0$
 $\varphi(\vec{zO}) = z' \varphi_0$

$\Leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ t_1 & \boxed{\varphi_0} \\ t_2 & \end{pmatrix}$

$\vec{\varphi} = \varphi$

45° Drehung um $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(21)



$$\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

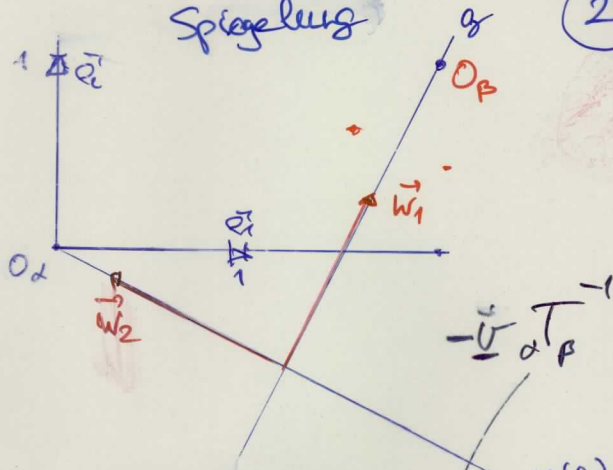
$(\varphi_P)_{\alpha}^{\beta} \tilde{T}_{\beta} \quad (\varphi_P)_{\beta}^{\alpha} \tilde{A} = \tilde{\varphi}_{O_{\beta}} \quad \tilde{T}_{\beta}^{\alpha} = \tilde{T}_{\beta}^{-1}$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ -2 & 1 & -1 \\ -4 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 3-\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 1-2\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad \varphi(O_{\alpha})^{\alpha} = \begin{pmatrix} 3-\sqrt{2} \\ 1-2\sqrt{2} \end{pmatrix}$$

$$\varphi_0^{\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Spiegelung



$$\tilde{T}_\beta \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1/\sqrt{5} & -2/\sqrt{5} \\ 1 & 2/\sqrt{5} & 1/\sqrt{5} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -4/\sqrt{5} & 1/\sqrt{5} & 2/\sqrt{5} \\ 3/\sqrt{5} & -2/\sqrt{5} & 1/\sqrt{5} \end{pmatrix}$$

$$= \tilde{T}_\beta \begin{pmatrix} 1 & 0 & 0 \\ -4/\sqrt{5} & 1/\sqrt{5} & 2/\sqrt{5} \\ 3/\sqrt{5} & -2/\sqrt{5} & 1/\sqrt{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 12/\sqrt{5} & -3/\sqrt{5} & 4/\sqrt{5} \\ -6/\sqrt{5} & 4/\sqrt{5} & 3/\sqrt{5} \end{pmatrix}$$

$$\varphi(O_\alpha) = \begin{pmatrix} 12/\sqrt{5} \\ -6/\sqrt{5} \end{pmatrix} \quad \varphi_{O_\alpha} = \begin{pmatrix} -3/\sqrt{5} & 4/\sqrt{5} \\ 4/\sqrt{5} & 3/\sqrt{5} \end{pmatrix}$$

$$\tilde{T}_\beta \tilde{U}_\beta \tilde{T}_\alpha = \tilde{U}_\alpha$$

gleit spiegellung

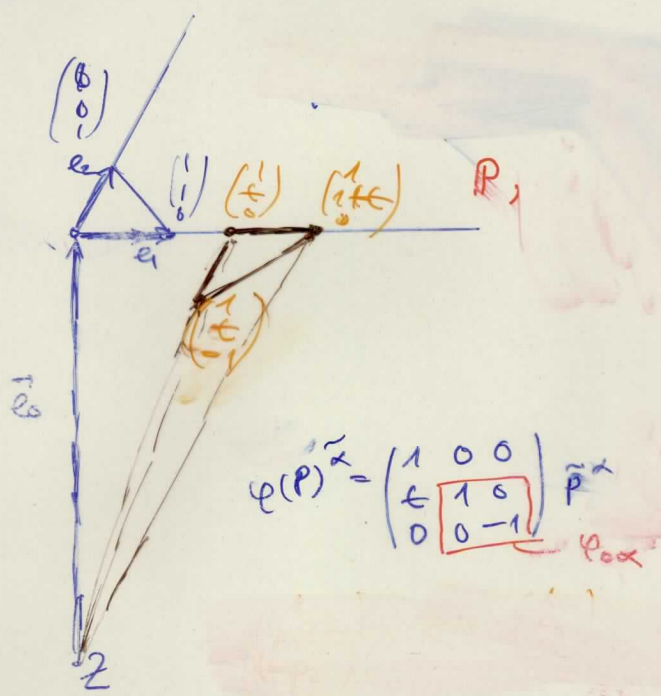
(23)

$$\varphi(P) = \vec{e} + \sigma(P), \quad \vec{e} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2/5 & -3/5 & 4/5 \\ 4/5 & 4/5 & 3/5 \end{pmatrix}$$

$$\varphi(O_2) = \begin{pmatrix} 12/5 \\ -6/5 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$\varphi_{0,2} = \begin{pmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{pmatrix}$$

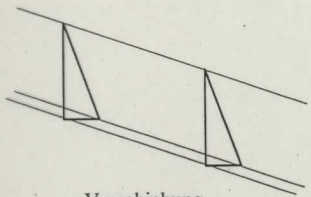


$\tilde{\varphi}: V \rightarrow V$ $\varphi(P) = \tilde{\varphi}(zP) + z$
 $P \in P$

14

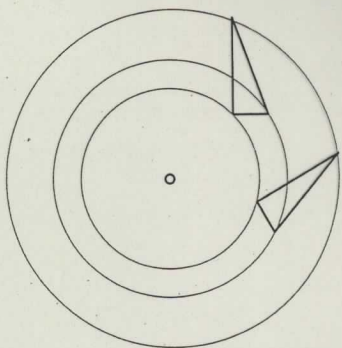
Bewegungen

$$|kP, \varphi Q| = |P, Q|$$



Verschiebung

translaciã



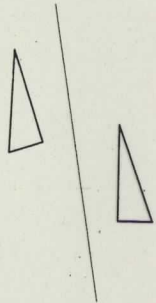
Drehung

rotaciã



Spiegelung

reflecciã



Gleitspiegelung

slide reflectiã