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Introductory Course Mathematics

Exercise Sheet 8

G34 Determine the tangent at x_0 :

(a) $f(x) = 2x^3 - 7, \quad x_0 = -1$

(b) $f(x) = \frac{1}{x}, \quad x_0 = \frac{1}{2}$

G35 Does $\lim\limits_{x \to x_0} \dfrac{f(x_0) - f(x)}{x_0 - x}$ exist for the following function?

$$f(x) = \left|x^3\right|, \quad x_0 = 0$$

Use the definition of differentiability to decide whether the function is differentiable in $x_0 = 0$.

G36 Prove from the definition of differentiability:

(a) If $f(x) = x^2$, then $f'(x) = 2x$.

(b) If $f(x) = x^3$, then $f'(x) = 3x^2$.

(c) If $f(x) = x^n$, for $n \in \mathbb{N}$, then $f'(x) = nx^{n-1}$.

(d) If $f(x) = \frac{1}{x}$, then $f'(x) = -\frac{1}{x^2}$.

G37 Write the following function as a composition of simpler functions and calculate their derivatives using the chain rule: $f(x) = \sqrt{(2x^2 + x)^3 + 1}$

G38 Prove using the defintion by power series from Lectures 6 and 7:

(a) If $f(x) = e^x$ then $f'(x) = e^x$.

(b) If $f(x) = \sin x$ then $f'(x) = \cos x$.

(c) If $f(x) = \cos x$ then $f'(x) = -\sin x$.

G39 Compute the derivatives of the following functions:

(a) $f_1(x) = x^4 - x^2 + 5x - 7$

(b) $f_2(x) = \frac{x^2+5}{\sqrt{x^2-7x+1}}$

(c) $f_3(x) = x^2 e^{x^2}$

(d) $f_4(x) = 2^x$

(e) $f_5(x) = x^x$

G40 Show, that $(f \pm g)' = f' \pm g'$.

G41 Use the product rule and the chain rule to prove the quotient rule.

G42 Decompose a fixed real number c into two summands such that their product is maximal.