



Introductory Course Mathematics

Exercise Sheet 7

G30 Determine the following limits if they exist.

(a) $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$

(c) $\lim_{x \rightarrow -1} \frac{x^4-1}{x+1}$

(b) $\lim_{x \rightarrow -\frac{1}{2}} \frac{4x^2-1}{2x+1}$

(d) $\lim_{x \rightarrow a} \frac{x^2-(a+1)x+a}{x^3-a^3}$

G31

(a) Consider the function $f : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto x$. Show that for all $c \in \mathbb{R}$ the limit $\lim_{x \rightarrow c} f(x)$ exists and that f is continuous on \mathbb{R} .

(b) For what values of the real number c is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} cx + 1 & \text{if } x \leq 3 \\ cx^2 - 1 & \text{if } x > 3 \end{cases}$$

continuous?

G32

(a) Use the Intermediate Value Theorem to show that there exists a positive number c such that $c^2 = 2$. (This proves the existence of the number $\sqrt{2}$.)

(b) Show that the following equations have a root in the given interval:

(i) $x^3 - 3x + 1 = 0$ in $[0, 1]$

(ii) $x^5 - 2x^4 - x - 3 = 0$ in $[2, 3]$

(iii) $x^2 = \sqrt{x+1}$ in $[1, 2]$

G33 Use the definition of continuity to prove Theorem 7.3.1:

Theorem 7.3.1. Let $f, g : U \rightarrow \mathbb{R}$ be two continuous functions. Then

- $f \pm g$,
- $f \cdot g$,
- $\frac{f}{g}$, and
- $f \circ g$

are continuous (where they are defined).