Fachbereich Mathematik
Dipl.-Math. Silke Möser

TECHNISCHE UNIVERSITÄT DARMSTADT

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Introductory Course Mathematics

Exercise Sheet 5

G19 (Limits I)

(a) Consider the sequence

$$a_n = \frac{2n-3}{5n+7}, \quad n \in \mathbb{N}.$$

(i) Show that the limit of this sequence is $\frac{2}{5}$.

(ii) Which terms of the sequence are closer to $\frac{2}{5}$ than $\varepsilon = \frac{1}{10}$?

(b) (i) What is the limit of the sequence $a_n = \frac{1}{2^n}$ for $n \in \mathbb{N}$?

(ii) What is the limit of the sequence

$$\frac{1}{2}, \quad \frac{1}{2}+\frac{1}{4}, \quad \frac{1}{2}+\frac{1}{4}+\frac{1}{8}, \quad \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}, \ldots$$

Can you give a geometric interpretation of this limit process?

(c) The first terms of an infinite sequence are $1, 3, 7, 15, 31, 63$.

(i) Find a recursive definition for the sequence.

(ii) Find an explicit definition.

(d) Find a recursive definition for the sequence

$$\sqrt{2}, \quad \sqrt{2\sqrt{2}}, \quad \sqrt{2\sqrt{2\sqrt{2}}}, \quad \ldots$$

What is the limit of this sequence?

G20 (Limits II)

Determine the limit (if it exists) of

$$a_n = \frac{5}{n} + \frac{7n}{n^2+1}, \qquad b_n = \left(6+\frac{1}{n}\right)\left(\frac{n+2}{2n+1}-1\right), \qquad c_n = \frac{2n^2-2}{3n^2-3},$$

$$d_n = \frac{\frac{1}{n^2}+\frac{1}{n^3}}{\frac{1}{n}+\frac{1}{n^2}}, \qquad e_n = \frac{2n+(-1)^n n}{n+1}.$$

G21 (Limits III)

Determine the limit (if it exists) of

(a) $a_n = \sqrt{n^2+1} - n, \quad n \in \mathbb{N}.$

(b) $b_n = n(\sqrt{n^2+1} - n), \quad n \in \mathbb{N}.$

(c) $c_n = n^2(\sqrt{n^2+1} - n), \quad n \in \mathbb{N}$.

G22 (The Fibonacci Sequence)

Consider the closed form for the Fibonacci sequence as given in the lecture:

$$f_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right).$$

(a) Prove that f_n is a natural number for $n = 1, 2, 3$.

(b) Prove that it is a natural number for every $n \in \mathbb{N}$.