Fachbereich Mathematik
Dipl.-Math. Silke Möser

TECHNISCHE
UNIVERSITÄT
DARMSTADT

WS 2010/11 30.9.2010

Introductory Course Mathematics

Exercise Sheet 4

G14 (Injectivity, Surjectivity, Bijectivity I)

(a) Which of the following functions are injective, surjective, bijective?

$$\begin{aligned} f_1 : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} f_2 : \mathbb{R}_{\geq 0} &\to \mathbb{R} \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} f_3 : \mathbb{R}_{\geq 0} &\to \mathbb{R}_{\geq 0} \\ x &\mapsto x^2 \end{aligned}$$

$$\begin{aligned} f_4 : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^3 \end{aligned}$$

$$\begin{aligned} f_5 : \mathbb{R} \setminus \{0\} &\to \mathbb{R} \\ x &\mapsto \frac{1}{x} \end{aligned}$$

$$\begin{aligned} f_6 : \mathbb{R} \setminus \{0\} &\to \mathbb{R}_{>0} \\ x &\mapsto \frac{1}{x^2} \end{aligned}$$

Also determine the image of each function.

(b) Find a function $f : \mathbb{N} \to \mathbb{N}$ which is

(i) injective but not surjective,

(ii) surjective but not injective.

G15 (Composition of Functions)

(a) Find functions f and g such that the following functions can be written as $f \circ g$.

$$F_1(x) = \sqrt{x+9} \qquad F_3(x) = \sqrt{x} + 2$$
$$F_2(x) = (x-5)^2 \qquad F_4(x) = \tfrac{1}{x-1}$$

(b) Consider the functions f and g from $\mathbb{R}$ to $\mathbb{R}$ given by $f(x) = x^2$ and $g(x) = x - 3$. Find the composite functions $f \circ f$, $f \circ g$, $g \circ f$ and $g \circ g$ and determine the domain of each function. Demonstrate that $f \circ g$ is not necessarily the same as $g \circ f$.

(c) Find $f \circ g \circ h$ where $f(x) = x/(x+1)$, $g(x) = x^2$ and $h(x) = x + 3$. Find the maximal subset of $\mathbb{R}$ on which $f \circ g \circ h$ is defined.

G16 (Preimages)

Determine the set

$$\{x \in \mathbb{R} \mid f(x) = 1\},$$

with

$$\begin{aligned} f : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^3 - x^2 - 4x + 5 \end{aligned}$$

G17 (Zeroes)

Find all zeroes of the following functions:

$$\begin{aligned} f : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^3 - 6x^2 + 11x - 6 \\ g : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^4 - 4x^3 + 6x^2 - 4x + 1 \\ h : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x^4 - 1 \end{aligned}$$

G18 (Bonus Exercise: Injectivity, Surjectivity, Bijectivity II) Let $f : X \to X$ be a function. Prove that f is

(a) injective,

(b) surjective,

(c) bijective

if and only if $f \circ f$ is.