Fachbereich Mathematik
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Introductory Course Mathematics

Exercise Sheet 2

G4 (Complex Numbers)

(a) Verify that the inversion formula

$$(a+bi)^{-1} = \frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i$$

is correct.

(b) Compute $\frac{5-3i}{3+2i}$.

(c) Try to find the solution of $(2-i)\cdot(2-2i)$ geometrically.

(d) Let $c = a+bi$ be a complex number. Compute $(x-c)(x-\bar{c})$. Can you guess a solution for $x^2-2x+2=0$?

G5 (Truth Tables)

Let A and B be propositions. Show that the following statments are true by setting up truth tables in each case:

(a) A is the same as $\neg(\neg A)$.

(b) $\neg(A \wedge B)$ is the same as $\neg A \vee \neg B$.

(c) $\neg(A \vee B)$ is the same as $\neg A \wedge \neg B$.

(d) $A \Rightarrow B$ is the same as $\neg A \vee B$.

G6 (Propositions and Quantifiers)

(a) Show: For propositions A and B we have that $A \wedge (A \Rightarrow B)$ implies B. Interpret this rule.

(b) Find examples of implications that are not equivalences and explain why the conclusion works in one direction only.

(c) Prove the proposition

$$(A \implies B) \iff (\neg B \implies \neg A)$$

(d) Negate the following propositions:

(i) All mathematicians are smokers.

(ii) No student likes to go to parties.

(iii) All bananas are yellow.

(iv) There exists a black swan.

(e) Negate the following propositions:

(i) $\exists x \in S : A(x) \wedge B(x)$

(ii) $\forall x \in S : A(x) \Rightarrow B(x)$

(f) Write the following propositions as a formal expression using quantifiers, negate the formal expression and convert the negation back into everyday language.

(i) For every real number x there exists a natural number n such that $n > x$.

(ii) There is no rational number x satisfying the equation $x^2 = 0$.