



Introductory Course Mathematics

Exercise Sheet 10

G48 Show that the union of two countable sets is countable.

G49 Show that any subset of a countable set is countable.

G50 Show that the set $\mathbb{N} \times \mathbb{N} = \{(m, n) \mid m, n \in \mathbb{N}\}$ of all pairs of natural numbers is countable. Deduce that the set of pairs $\{(x, y) \mid x \in X, y \in Y\}$ is countable if X and Y are both countable sets.

G51 Show that the set of all infinite sequences consisting of 0s and 1s is uncountable.

G52 Reexamine Cantor's first diagonal process for the countability of the rational numbers. How often does it list each rational number?

G53 A rational number can also be written as an infinite decimal expansion. Try to apply Cantor's second diagonal process to the set of rational numbers and try to show that the set of rational numbers is uncountable (which is false, of course). Where does the argument break down?