Model Theory

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Exercises No.7

Exercise 1 [universality and saturation]
Find an example of a complete countable theory T s.t.

(a) T has a countable ω-saturated model,

(b) T also has a countable model that is universal w.r.t. elementary embeddings within the class of all countable models of T but fails to be ω-saturated.

Hint: the desired model must fail to be ω-homogeneous. You may want to think of theories whose model class is closed under disjoint sums; note that the universality condition is preserved under elementary extensions (why?), while ω-saturation or ω-homogeneity typically are not.

Exercise 2 [discrete linear orderings]
Let T be the theory of discrete linear orderings $<$ with first and without last element. Show that T is complete and discuss its type spaces. Does T have atomic models, countable ω-saturated models? Determine all countable ω-homogeneous models of T.

Exercise 3 [asymptotic probabilities and the random graph]
Determine the asymptotic probability (or its non-existence, as the case may be) of the following properties of finite graphs.

(i) even cardinality

(ii) existence of at least one isolated node

(iii) connectedness

(iv) diameter 2

(v) planarity

(vi) k-colourability ($k > 1$ fixed)

(vii) triangle-freeness

(viii) existence of at least two nodes of the same degree

(ix) existence of a node of degree 17

Exercise 4 [Fraïssé limits]
Which of the following classes K_i of finite structures possess a Fraïssé limit? Determine the limit structure and discuss its theory.

(i) K_1: finite graphs without n-cliques (for fixed $n \geqslant 3$)

(ii) K_2: finite planar graphs

(iii) K_3: finite bipartite graphs (in signature $\tau = \{E\}$)

(iv) K_4: finite bipartite coloured graphs (in signature $\tau = \{E, P\}$)

(v) K_5: finite equivalence relations

(vi) K_6: finite linear orderings

(vii) K_7: acyclic finite graphs