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Höhere Mathematik 2

3. Übung, Lösungsvorschlag

Gruppenübungen

Aufgabe G7 Es ist

$$h(t) = (g \circ f)(t) = g(f(t)) = g(e^t, t^2) = (e^t)^3 + 8(t^2)^3 - 3e^t t^2 = e^{3t} + 8t^6 - 3t^2 e^t.$$

Damit gilt

$$Dh(t) = h'(t) = 3e^{3t} + 48t^5 - 6te^t - 3t^2 e^t = 3e^{3t} + 48t^5 - e^t(3t^2 + 6t)$$

und außerdem

$$\nabla g(f(t)) \cdot Df(t) = \nabla g(e^t, t^2) \cdot \begin{pmatrix} e^t \\ 2t \end{pmatrix}.$$

Es gilt $\nabla g(x,y) = (3x^2 - 3y, 24y^2 - 3x)$, also ist

$$\begin{aligned} \nabla g(f(t)) \cdot Df(t) &= (3(e^t)^2 - 3t^2, 24(t^2)^2 - 3e^t) \cdot \begin{pmatrix} e^t \\ 2t \end{pmatrix} \\ &= 3(e^t)^3 - 3t^2 e^t + 48t^5 - 6te^t = 3e^{3t} + 48t^5 - e^t(3t^2 - 6t). \end{aligned}$$

Aufgabe G8 Für Matrix A gilt:

$$\begin{aligned} & \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right) \quad +\text{Zeile 1} \\ \rightsquigarrow & \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right) \quad \cdot\tfrac{1}{2} \\ \rightsquigarrow & \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array}\right) \quad -\text{Zeile 2} \\ \rightsquigarrow & \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array}\right). \end{aligned}$$

Daraus folgt

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Für Matrix B gilt:

$$\left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} \\ -\text{Zeile } 1 \\ -\text{Zeile } 1 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right) \quad \begin{array}{l} \\ \\ \cdot(-\frac{1}{2}) \end{array}$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right) \quad \text{Zeile } 2 \leftrightarrow \text{Zeile } 3$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -1 & 1 & -1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} -4 \cdot \text{Zeile } 2 \\ \\ +\text{Zeile } 2 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & -1 & 0 & 2 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{array}\right) \quad \begin{array}{l} -3 \cdot \text{Zeile } 3 \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -3 & \frac{7}{2} \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{array}\right).$$

Daraus folgt

$$B^{-1} = \begin{pmatrix} \frac{1}{2} & -3 & \frac{7}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -6 & 7 \\ 1 & 0 & -1 \\ -1 & 2 & -1 \end{pmatrix}.$$

Aufgabe G9

$$\det A = \begin{vmatrix} 3 & 2 \\ 5 & 7 \end{vmatrix} = 3 \cdot 7 - 5 \cdot 2 = 11.$$

$$\det B = \begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & -5 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot 2 \cdot 1 = 2.$$

Hausübungen

Aufgabe H7

a)

$$h(u,v) = (g \circ f)(u,v) = (e^{u+v}\cos(e^{u-v}), e^{u+v}\sin(e^{u-v}).$$

$$J_f(u,v) = \begin{pmatrix} e^{u+v} & e^{u+v} \\ e^{u-v} & -e^{u-v} \end{pmatrix}.$$

$$J_g(r,\theta) = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}.$$

Nach der Kettenregel gilt

$$\begin{aligned} J_h(u,v) &= J_g(f(u,v))J_f(u,v) \\ &= \begin{pmatrix} \cos(e^{u-v}) & -e^{u+v}\sin(e^{u-v}) \\ \sin(e^{u-v}) & e^{u+v}\cos(e^{u-v}) \end{pmatrix} \begin{pmatrix} e^{u+v} & e^{u+v} \\ e^{u-v} & -e^{u-v} \end{pmatrix} \\ &= \begin{pmatrix} e^{u+v}\cos(e^{u-v}) - e^{u-v}e^{u+v}\sin(e^{u-v}) & e^{u+v}\cos(e^{u-v}) + e^{u-v}e^{u+v}\sin(e^{u-v}) \\ e^{u+v}\sin(e^{u-v}) + e^{u-v}e^{u+v}\cos(e^{u-v}) & e^{u+v}\sin(e^{u-v}) - e^{u-v}e^{u+v}\cos(e^{u-v}) \end{pmatrix}. \end{aligned}$$

b)

$$h(t) = (g \circ f)(t) = \cos^2 t + \sin^2 t + e^{2t} = 1 + e^{2t}.$$
$$Dh(t) = h'(t) = 2e^{2t}.$$

Aufgabe H8 Es gilt:

$$\left(\begin{array}{rrrr|rrrr} 1 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ -1 & 2 & 1 & -2 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 4 & 0 & 0 & 1 & 0 \\ 3 & -2 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ +\text{Zeile } 1 \\ -2 \cdot \text{Zeile } 1 \\ -3 \cdot \text{Zeile } 1 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{rrrr|rrrr} 1 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -6 & 0 & -2 & 0 & 1 & 0 \\ 0 & -2 & -8 & -4 & -3 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ \cdot 2 - \text{Zeile } 2 \\ +\text{Zeile } 2 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{rrrr|rrrr} 1 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -16 & 0 & -5 & -1 & 2 & 0 \\ 0 & 0 & -4 & -4 & -2 & 1 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ \\ \cdot 4 + \text{Zeile } 3 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{rrrr|rrrr} 1 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -16 & 0 & -5 & -1 & 2 & 0 \\ 0 & 0 & 0 & -16 & -3 & 5 & -2 & 4 \end{array}\right) \begin{array}{l} \\ \cdot\frac{1}{2} \\ \cdot(-\frac{1}{16}) \\ \cdot(-\frac{1}{16}) \end{array}$$

$$\rightsquigarrow \left(\begin{array}{rrrr|rrrr} 1 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{5}{16} & \frac{1}{16} & -\frac{2}{16} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{16} & -\frac{5}{16} & \frac{2}{16} & -\frac{4}{16} \end{array}\right) \begin{array}{l} -3 \cdot \text{Zeile } 3 \\ -2 \cdot \text{Zeile } 3 \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left(\begin{array}{rrrr|rrrr} 1 & 0 & 0 & 2 & \frac{1}{16} & -\frac{3}{16} & \frac{6}{16} & 0 \\ 0 & 1 & 0 & 0 & -\frac{2}{16} & \frac{6}{16} & \frac{4}{16} & 0 \\ 0 & 0 & 1 & 0 & \frac{5}{16} & \frac{1}{16} & -\frac{2}{16} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{16} & -\frac{5}{16} & \frac{2}{16} & -\frac{4}{16} \end{array}\right) \begin{array}{l} -2 \cdot \text{Zeile } 4 \\ \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left(\begin{array}{rrrr|rrrr} 1 & 0 & 0 & 0 & -\frac{5}{16} & \frac{7}{16} & \frac{2}{16} & \frac{8}{16} \\ 0 & 1 & 0 & 0 & -\frac{2}{16} & \frac{6}{16} & \frac{4}{16} & 0 \\ 0 & 0 & 1 & 0 & \frac{5}{16} & \frac{1}{16} & -\frac{2}{16} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{16} & -\frac{5}{16} & \frac{2}{16} & -\frac{4}{16} \end{array}\right).$$

Daraus folgt

$$A^{-1} = \frac{1}{16} \begin{pmatrix} -5 & 7 & 2 & 8 \\ -2 & 6 & 4 & 0 \\ 5 & 1 & -2 & 0 \\ 3 & -5 & 2 & -4 \end{pmatrix}.$$

Aufgabe H9

$$\det A = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 6 & 9 \end{vmatrix} = 1 \cdot 4 \cdot 9 = 36.$$

$$
\begin{aligned}
\det B &= \begin{vmatrix} 2 & 3 & 4 \\ 0 & 0 & -1 \\ 5 & 6 & 7 \end{vmatrix} \\
&= (-1)^0 \cdot 2 \cdot \begin{vmatrix} 0 & -1 \\ 6 & 7 \end{vmatrix} + (-1)^1 \cdot 3 \cdot \begin{vmatrix} 0 & -1 \\ 5 & 7 \end{vmatrix} + (-1)^2 \cdot 4 \cdot \begin{vmatrix} 0 & 0 \\ 5 & 6 \end{vmatrix} \\
&= 2 \cdot 6 - 3 \cdot 5 + 4 \cdot 0 = -3.
\end{aligned}
$$