Fachbereich Mathematik
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7. Problems for CMC Surfaces

Problem 23 – Continuous functions of surfaces:

Give the definition of a continuous function from a surface to a surface.

Problem 24 – Constant mean curvature surfaces bounded by circles:

a) Suppose $M \subset \mathbb{R}^3$ be an embedded surface
- with mean curvature 1,
- the boundary ∂M is a circle of radius $R > 1$,
- M is contained in a halfspace (determined by the plane of the circle).

However, we do not assume that M is bounded. Prove that M cannot exist.
Hint: You can assume that such an M would decompose the closed upper halfspace $\{z \geq 0\}$ into two connected components U and V.

b) Suppose $M \subset \mathbb{R}^3$ is a surface
- with mean curvature 1,
- M is a bounded,
- the boundary of M is a circle of radius $R \leq 1$,
- M is contained in a halfspace (determined by the plane of the circle).

Prove that (as stated in class) Alexandrov reflection works to show that M is a spherical cap of a unit sphere.

c) Generalize the statements to arbitrary dimension – are they true?

Problem 25 – Alexandrov embedding in dimension 2:

Consider a map $\Phi \in C^0(\overline{D}, \mathbb{R}^2)$, which is an immersion on D; let $\varphi := \Phi|_{\mathbb{S}^1}$ be the boundary restriction.

a) Suppose $\varphi\colon \mathbb{S}^1 \to \mathbb{R}^2$ is injective. Show that there is a unique immersion Φ extending φ to D, up to diffeomorphism.
Hint: Consider the number of preimages of Φ on $\mathbb{R}^2 \setminus \varphi(\mathbb{S}^1)$.

b) We show that the extension Φ of φ if φ is not injective. That is, there exist immersions Φ, Ψ of the disk with continuous boundary values $\Psi|_{\mathbb{S}^1} = \Phi|_{\mathbb{S}^1} = \varphi$, but there is no diffeomorphism $\sigma\colon D \to D$, such that $\Psi \circ \sigma = \Phi$.
To see that, find a mirror symmetric polygon in $\mathbb{R}^2$ with self-intersections, for which the extension Φ is not be mirror symmetric. It is sufficient to consider a hexagon (or pentagon).