Fachbereich Mathematik
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TECHNISCHE
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11. Problems for CMC Surfaces

Problem 39 – Quiz:

True or false?

a) A constant mean curvature surface with boundary and compact closure is always contained in the convex hull of its boundary values.

b) The matrix of the shape operator S has symmetric entries.

c) If two surfaces of constant mean curvature are one-sided tangent at an interior point then they agree locally.

d) There is no graph over the entire plane $\mathbb{R}^2$ with mean curvature $H > \epsilon$. Here, determine the assumption on ϵ you need to make this true. Give the answer both for H constant and H variable.

Problem 40 – Theorems:

What are the main results of the lecture?

Problem 41 – Upper semicontinuity:

a) Give a simple example of an upper semicontinuous function from $\mathbb{R}$ to $\mathbb{R}$, and one with many discontinuities.

b) Given a monotone function on $\mathbb{R}$, prove that the number of discontinuities is countable.

c) Prove that an upper semincontinuous function takes a maximum on each compact subset $K \subset \mathbb{R}$.

Problem 42 – Alexandrov reflection for minimal surfaces:

Prove that a properly embedded minimal surface with two ends $\varphi\colon \Sigma_g \setminus \{p_1, p_2\} \to \mathbb{R}^3$, such that the two ends are asymptotically catenoids must be a catenoid. You can assume convergence of the surface together with its normal when $|x| \to \infty$. In particular, Σ must have genus $g = 0$.

a) Use force balancing to show that the axes of the two catenoid ends are parallel (you might as well assume this property). By the way, what does this imply for the growth rates?

b) Apply Alexandrov reflection.

Remarks: 1. The asymptotics to catenoids is a consequence of the Weierstrass representation formulas.
2. The statement was first proved by Rick Schoen (1983).

Problem 43 – Alexandrov embedded:

Prove that the nodoid is not Alexandrov embedded, that is, there is no immersion $F\colon B^3 \setminus \{(0,0,\pm1)\} \to \mathbb{R}^3$, whose restriction to the boundary $\mathbb{S}^2 \setminus \{(0,0,\pm1)\}$, parameterizes a nodoid.
Hint: Apply the Gauss-Bonnet formula to a suitable planar slice.