Fachbereich Mathematik
Prof. Dr. W. Trebels
Dr. V. Gregoriades
Dr. A. Linshaw

TECHNISCHE UNIVERSITÄT DARMSTADT

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13th Homework Sheet Analysis II (engl.)
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(H13.1)

1. Prove that if f is a C^2-function from $\mathbb{R}^3$ to $\mathbb{R}$ then $\nabla \times (\nabla f) = 0$. (This is property (c) after Definition 2.3 Chap. IX for C^2 functions instead of scalar C^1 functions).
2. Is the vector function $F = (F_1, F_2, F_3) : \mathbb{R}^3 \to \mathbb{R}^3 : F(x,y,z) = (y, -x, 0)$ a gradient field?

(H13.2)

1. Compute the integral
$$I := \int_{-1}^{1} \int_{2}^{3} \frac{y}{\log(x)} dx dy.$$
2. Let $R := [a,b] \times [c,d]$ and $f : R \to \mathbb{R}$ be a continuous function. For $a < x < b$ and $c < y < d$, we define
$$F(x,y) := \int_a^x \int_c^y f(t,s) ds dt.$$
Prove that $\dfrac{\partial^2 F}{\partial x \partial y} = \dfrac{\partial^2 F}{\partial y \partial x}$.

(H13.3)

1. Which of the following sets have Lebesgue measure zero?
$$\mathbb{Q},\ (0,1) \times (0,1),\ \text{the unit circle } C := \{(x,y) \in \mathbb{R}^2 \ \mid \ x^2 + y^2 = 1\}.$$
Hint. For the set $(0,1) \times (0,1)$ you may take for granted that if $[a,b] \times [c,d] \subseteq \cup_{i=1}^n R_i$ where R_i are rectangles, then $\sum_{i=1}^n |R_i| \geq (b-a)(d-c)$.
2. Give the example of a function $f : \mathbb{R} \to \mathbb{R}$ which is discontinuous at infinitely many $x \in \mathbb{R}$ but it is almost everywhere continuous. *Hint.* Look for such an example in Section 1 of Chapter III of the Script.