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# 10th Homework Sheet Analysis II (engl.)

## Summer Semester 2010

**(H10.1)**

Consider the surface

$$S := \{(x, y, z) : x^3 + 3y^2 + 8xz^2 - 3z^3y = 1\}$$

and a point $(x_0, y_0, z_0) \in S$. Give conditions for the triple $(x_0, y_0, z_0)$ under which there is an open set $W \subseteq \mathbb{R}^2$ which contains $(x_0, y_0)$ and there is a differentiable function $\varphi : W \to \mathbb{R}$ such that $\varphi(x_0, y_0) = z_0$ and $(x, y, \varphi(x, y)) \in S$ for all $(x, y) \in S$.

*Advice.* When we say "give conditions for $(x_0, y_0, z_0)$" it is enough to say for example that $z_0 \neq x_0 y_0$; (look also the hint of H9.3 and the solution of G10.1).

**(H10.2)**

Prove that there is some open set $W \subseteq \mathbb{R}^2$ such that $(1, 1) \in W$ and that there are differentiable functions $u \equiv u(x, y)$ and $v \equiv v(x, y)$ defined on $W$ with $u(1, 1) = v(1, 1) = 1$ which satisfy the following system of equations

$$\begin{aligned} xu + yvu^2 &= 2, \\ xu^3 + y^2v^4 &= 2. \end{aligned}$$

Compute the partial derivative $\dfrac{\partial u}{\partial x}(1, 1)$.

**(H10.3)**

Define the sets

$$A = \{(x,y,z) \in \mathbb{R}^3 : z^2 + xy - 1 = 0\}, B = \{(x,y,z) \in \mathbb{R}^3 : z^2 + x^2 - y^2 - 1 = 0\}$$

and suppose that $(x_0, y_0, z_0) \in A \cap B$. Find sufficient conditions for $(x_0, y_0)$ under which there is an open $W \subseteq \mathbb{R}$ with $z_0 \in W$ and differentiable functions $f, g : W \to \mathbb{R}$ such that $f(z_0) = x_0$, $g(z_0) = y_0$ and $(f(z), g(z), z) \in A \cap B$ for all $z \in W$. Compute the derivatives $f'(z)$ and $g'(z)$ with respect to $z, f(z), g(z)$, for all $z \in W$.

*Hint.* Treat the pair $(x, y)$ as a function of $z$. You need to apply the Implicit Function Theorem (Theorem 2.1 Chap. VIII) to some function $F(z, x, y) = (F_1(z, x, y), F_2(z, x, y))$.