Fachbereich Mathematik
Prof. Dr. W. Trebels
Dr. V. Gregoriades
Dr. A. Linshaw

TECHNISCHE UNIVERSITÄT DARMSTADT

02-06-2010

8th Homework Sheet Analysis II (engl.)

Summer Semester 2010

(H8.1) Let $f(x,y) = x^3 + y^2 - 3x - 4y + 1$, and let D be the rectangle $\{(x,y)|\ 0 \leq x \leq 3,\ 0 \leq y \leq 3\}$ whose vertices are $(0,0)$, $(3,0)$, $(0,3)$ and $(3,3)$.

1. Find all critical points of f inside D.
2. Find the extreme values of f along the boundary of D. You need to consider the values of f along each line segment and at each vertex of the boundary of D.
3. Find the global maximum and minimum values of f on D and indicate the point(s) in D where these values are achieved.

(H8.2)

1. Find a function $f(x,y)$ such that $(1,2)$ is a critical point, $H_f(1,2) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, and f achieves a global maximum at $(1,2)$.
2. Find a function $g(x,y)$ with a critical point at $(1,2)$ with $H_g(1,2) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, such that g has no local extremum at $(1,2)$.
3. Find a function $h(x,y)$ which is differentiable on all of $\mathbb{R}^2$, which has a local minimum at every point along the circle $x^2 + y^2 = 1$.

(H8.3)
Define $f(0,0) = 0$, and

$$f(x, y) = x^2 + y^2 - 2x^2 y - \frac{4x^6 y^2}{(x^4 + y^2)^2}, \qquad (x, y) \neq (0, 0).$$

1. Prove that for all $(x, y) \in \mathbb{R}^2$, $4x^4 y^2 \leq (x^4 + y^2)^2$. Conclude that f is continuous on all of $\mathbb{R}^2$.

2. For $0 < \theta < 2\pi$, $t \in \mathbb{R}$, define $g_\theta(t) = f(t\cos\theta, t\sin\theta)$. Show that $g_\theta(0) = 0$, $g'_\theta(0) = 0$, and $g''_\theta(0) = 2$. Each g_θ therefore has a strict local minimum at $t = 0$. In other words, the restriction of f to each line through $(0, 0)$ has a local minimum at $(0, 0)$.

3. Show that $(0, 0)$ is not a local minimum for f, since $f(x, x^2) = -x^4$.