Fachbereich Mathematik
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5th Homework Sheet Analysis II (engl.)

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(H5.1)

1. Find sets $A_i, B_i \subseteq \mathbb{R}^2$, $i = 1, 2$ such that:
   (a) A_1, B_1 are connected and $A_1 \cup B_1$ is not connected.
   (b) A_2, B_2 are connected and $A_2 \cap B_2$ is not connected, (and also $A_2 \cap B_2 \neq \emptyset$).

   It is enough to give a sketch of those sets.
2. Prove that a metric space M is connected if and only if for all $A \subseteq M$ such that A is open and closed we have that either $A = \emptyset$ or $A = M$.

(H5.2)

1. Compute $\mathrm{grad} f(x_0, y_0, z_0)$ for arbitrary $x_0, y_0, z_0 \in \mathbb{R}$ where f is given by $f(x, y, z) = x \cdot e^{-x^2-y^2-z^2}$, $x, y, z \in \mathbb{R}$.
2. Given $u = (1/\sqrt{3}, 1\sqrt{3}, 1/\sqrt{3}) \in \mathbb{R}^3$ and $f(x, y, z) = z^2x + y^3$, $x, y, z \in \mathbb{R}$, compute the directional derivative $D_u f(1, 1, 2)$.
3. Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ which is defined as follows:
   $f(x, y) = (x^2 + y^2) \cdot \sin(\frac{1}{\sqrt{x^2+y^2}})$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Prove the following.
   (a) The partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist on every point $(x_0, y_0) \in \mathbb{R}^2$.
   (b) The partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ are not continuous.
   (c) The function f is differentiable.

(H5.3)

Define the function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(x, y) = \dfrac{y^3}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Prove that (a) the function f is continuous, (b) every directional derivative $D_u f(0, 0)$ (for $u \in \mathbb{R}^2$ with $\|u\|_2 = 1$) exists and (c) the function f is not differentiable at $(0, 0)$.