Fachbereich Mathematik
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1st Homework Sheet Analysis II (engl.)
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(H1.1)

Compute the following integrals.

1. $\int_0^1 x^2 \cdot e^x dx$
2. $\int_0^{1/2} \frac{x^2}{x^2-1} dx$
3. $\int_0^2 \frac{1}{x^2+4} dx$

(H1.2)

1. Suppose that $f : [0, +\infty) \to \mathbb{R}$ is a periodic function with period $a > 0$ and that f is jump continuous on $[0, a]$ - notice that f must be jump continuous on every closed interval I. Prove that for all $k \in \mathbb{N}$ we have that $\int_0^a f(x)dx = \int_{ka}^{(k+1)a} f(x)dx$. Conclude that $\int_0^a f(x)dx = \int_b^{b+a} f(x)dx$ for all $b > 0$.
2. Find all continuous functions $g : [1, \infty) \to \mathbb{R}$ such that the function h_g defined by $h_g(x) := \int_1^x t \cdot g(t)dt - (x + x^2)$, $x \in [1, \infty)$, is constant. What are the possible values of any such h_g?

(H1.3)

1. Suppose that the functions $g, h : \mathbb{R} \to \mathbb{R}$ are differentiable and that the function $f : \mathbb{R} \to \mathbb{R}$ is continuous. Prove that the function $F : \mathbb{R} \to \mathbb{R}$ which is defined by $F(x) = \int_{g(x)}^{h(x)} f(t)dt$ satisfies that $F'(x) = f(h(x)) \cdot h'(x) - f(g(x)) \cdot g'(x)$ for all $x \in \mathbb{R}$. *Hint.* It is true that $\int_{g(x)}^{h(x)} f(t)dt = \int_0^{h(x)} f(t)dt - \int_0^{g(x)} f(t)dt$. Then refer to (1) of G1.3.
2. Compute the limit $\lim_{x \to 0^+} x \cdot \int_x^1 \frac{e^t}{t} dt$. *Hint.* Try to bound only the quantity e^t, for $x \le t \le 1$.