Fachbereich Mathematik
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13th Tutorial Analysis II (engl.)

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(T13.1) (Fubini's Theorem not applicable)
Let $f : [0,1] \times [0,1] \to \mathbb{R}$,

$$f(x,y) = \begin{cases} \dfrac{x-y}{(x+y)^3} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Compute $I_1 = \int_0^1 \left(\int_0^1 f(x,y)\,dx \right) dy$ and $I_2 = \int_0^1 \left(\int_0^1 f(x,y)\,dy \right) dx$.

(b) Remark that $I_1 \neq I_2$. Does this contradict Fubini's Theorem?

(T13.2) (Fubini's Theorem not Applicable).
On the square $Q := [0,1] \times [0,1]$ we consider the function $f\colon Q \to \mathbb{R}$,

$$f(x,y) := \begin{cases} 0 & \text{if } x \text{ or } y \text{ are irrational;} \\ 1/n & \text{if } y \text{ is rational and } x = m/n \\ & \text{with } n \in \mathbb{N},\ m \in \mathbb{N}_0 \text{ relatively prime.} \end{cases}$$

(a) Show that $\int_0^1 f(x,y)\,dx$ exists and has the value 0 for every $y \in [0,1]$.

(b) Show that the following integrals exist and both vanish:

$$\int_0^1 \left[\int_0^1 f(x,y)\,dx \right] dy = \int_Q f(x,y)\,d(x,y) = 0\,.$$

(c) Show that $\int_0^1 f(x,y)\,dy$ does not exist for rational $x \in [0,1]$.

(T13.3) (The Center of Gravity of a Pyramid)

We consider a pyramid P with a rectangular base Q which is parallel to the (x, y)-plane and which is given by letting the vertices be the points (a, b, c), (a, B, c), (A, b, c) and (A, B, c) with $a < A$, $b < B$ and $c > 0$. Let the apex of the pyramid be placed in the origin $(0, 0, 0)$. Notice that this pyramid is placed "upside down".

The *center of gravity* of P is the point $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ with

$$\bar{x}_i = \frac{1}{|P|} \int_P x_i \, dx.$$

Calculate the center of gravity of this pyramid.