Fachbereich Mathematik
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6th Tutorial Analysis II (engl.)

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(T6.1)

1. Prove that any path-connected metric space is connected. (This is Theorem 4.7 Chap. VII).

2. This is a standard example of a connected topological space which is not path-connected. Define

$$X_1 = \{0\} \times [-1, 1], \quad X_2 = \{(s, \sin(\frac{1}{s})) \in \mathbb{R}^2 \ \mid \ s > 0\}, \quad \text{and} \quad X = X_1 \cup X_2.$$

   We think of X with the metric induced by the standard metric on $\mathbb{R}^2$. (The metric space X_2 is called the "topologist's sine curve", and X is the closure of X_2 in $\mathbb{R}^2$.)

   Prove that X is connected but not path-connected.

(T6.2)

Let W be a normed space over $\mathbb{K}$, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, and consider $\mathbb{K}^n$ equipped with the supremum norm $\| \cdot \|_\infty$. Prove that any linear transformation $T : \mathbb{K}^n \to W$ is continuous.

(T6.3)

Prove Corollary 2.5 Chap. VII from the lecture:
Let $J \subseteq \mathbb{R}$ be an interval, let $V \subseteq \mathbb{R}^m$ be open, and let $f = (f_1, \dots, f_m) : J \to V$ and $g : V \to \mathbb{R}$ be differentiable functions. Then $g \circ f : J \to \mathbb{R}$ is differentiable, and for all $x_0 \in J^\circ$ we have

$$(g \circ f)'(x_0) = D(g \circ f)(x_0) = \langle \operatorname{grad} g(f(x_0)),\ f'(x_0) \rangle = \sum_{j=1}^{m} \frac{\partial g}{\partial x_j}(f(x_0)) f_j'(x_0).$$