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1st Tutorial Analysis II (engl.)

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(T1.1)

Compute using the definition the integral $\int_0^1 x^2 dx$.

Hint: Choose easy step functions φ_n for all $n \in \mathbb{N}$ such that the sequence (φ_n) converges uniformly to f. Also recall that $\sum_{k=1}^m k^2 = \frac{m(m+1)(2m+1)}{6}$.

We proceed with some definitions. Let a function $f : [a, b] \to \mathbb{R}$ and a partition $P = (t_0, \dots, t_n)$ of $[a, b]$. We define $M_i := \sup\{f(x) \ / \ x \in [t_i, t_{i+1}]\}$ and $m_i := \inf\{f(x) \ / \ x \in [t_i, t_{i+1}]\}$ for all $i = 0, \dots, n-1$. Define also

$$U(f, P) = \sum_{i=0}^{n-1} M_i \cdot (t_{i+1} - t_i) \quad \text{and} \quad L(f, P) = \sum_{i=0}^{n-1} m_i \cdot (t_{i+1} - t_i).$$

(T1.2)

Suppose that we are given a function $f : [a, b] \to \mathbb{R}$ and partitions $P = (t_0, \dots, t_n)$, $Q = (s_0, \dots, s_m)$ of $[a, b]$ such that $\{t_0, \dots, t_n\} \subseteq \{s_0, \dots, s_m\}$. Prove that

$$L(f, P) \le L(f, Q) \le U(f, Q) \le U(f, P).$$

(T1.3)

Suppose that the function $f : [a, b] \to \mathbb{R}$ is jump continuous; prove that

$$\begin{aligned} \int_a^b f(x)dx &= \inf\{U(f, P) \ / \ P \text{ is a partition of } [a, b]\} \\ &= \sup\{L(f, P) \ / \ P \text{ is a partition of } [a, b]\}. \end{aligned}$$

Hint. Use Theorem 2.16 Chap. 5.