Fachbereich Mathematik
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14th Exercise Sheet Analysis II (engl.)
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(G14.1)

1. Let $a > 0$. Find the volume of the solid A lying in the first octant of $\mathbb{R}^3$ (i.e., the set of (x, y, z) with $x \geq 0$, $y \geq 0$, $z \geq 0$), inside the cylinder given by $x^2 + y^2 = a^2$, and under the plane given by $y = z$.

2. Let $a > 0$. Find the volume of the subset A of $\mathbb{R}^3$ lying inside both the sphere given by $x^2 + y^2 + z^2 = 4a^2$ and the cylinder given by $x^2 + y^2 = 2ay$.

(G14.2) Let $a, b > 0$. Use a suitable change of variables to calculate the area of the elliptic disk E in $\mathbb{R}^2$ given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1.$$

(G14.3) Consider the integral of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ over the "ice cream cone" shaped region D bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere of radius 2.

1. Write down (but do not evaluate) this integral in the form

$$\int_a^b \int_{\psi_1(x)}^{\psi_2(x)} \int_{\gamma_1(x,y)}^{\gamma_2(x,y)} f(x, y, z)\; dz\; dy\; dx.$$

2. Describe the region D in spherical coordinates.

3. Rewrite the above integral in spherical coordinates and evaluate it.