

Fachbereich Mathematik
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02-06-2010

8th Exercise Sheet Analysis II (engl.) Summer Semester 2010

(G8.1) (Extrema on compact sets)

Let $K = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. Find the local and global extrema of the function

$$f : K \rightarrow \mathbb{R}, \quad f(x, y) = x^2 y.$$

Hint: To compute the global extrema of a function f defined on a compact subset K of \mathbb{R}^n , you have to compute the local extrema on the interior of K and on the boundary of K .

(G8.2) Consider the integral $I(y) = \int_0^\pi e^{-x^2} \cos(2xy) dx$, $y \in \mathbb{R}$. Show that for all $k \in \mathbb{Z}$,

$$I'(k) + 2kI(k) = 0.$$

(G8.3) (Least square method)

Consider points $(x_1, y_1), \dots, (x_k, y_k) \in \mathbb{R}^2$, where not all x_i are identical. Determine a straight line $y = mx + b$ such that the square distance

$$F(m, b) = \sum_{i=1}^k (mx_i + b - y_i)^2$$

becomes a minimum.