Fachbereich Mathematik
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20-05-2010

6th Exercise Sheet Analysis II (engl.)

Summer Semester 2010

(G6.1)

Prove Corollary 2.10 Chap. VII:
Let $U \subseteq \mathbb{R}^n$ be an open set such that for any two elements $x, y \in U$ there exist points $x = z_0, z_1, \ldots, z_l = y$ such that $\overline{z_{k-1} z_k} \subseteq U$ for all $k = 1, \ldots, l$. Let $f : U \to \mathbb{R}$ be differentiable. Then f is constant if and only if $\operatorname{grad} f(x) = 0$ holds for all $x \in U$.

(G6.2)

Define the function $f : \mathbb{R}^2 \to \mathbb{R}$ as follows: $f(x, y) = \dfrac{x^3 y - x y^3}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Prove that $f \in C^1(\mathbb{R}^2, \mathbb{R})$, the partial derivatives f_{xy} and f_{yx} exist everywhere in $\mathbb{R}^2$ and are continuous on $\mathbb{R}^2 \setminus \{(0, 0)\}$, but $f_{xy}(0, 0) = 1$ and $f_{yx}(0, 0) = -1$. (This is example 3.1 Chap. VII).

(G6.3)

Let $A = (a_{ij})_{i,j=1,\ldots,n}$ be a symmetric real $n \times n$-matrix and let

$$F : \mathbb{R}^n \to \mathbb{R}, \quad F(x) = x^T A x = \sum_{i,j=1}^{n} a_{ij} x_i x_j.$$

We saw in Example 1.3 Chap. VII that $F'(x) = (2Ax)^T$. Compute the Hessian of F.