Fachbereich Mathematik
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2rd Exercise Sheet Analysis II (engl.)
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(G2.1)

1. For $x, y \in \mathbb{R}$, define the following functions : $d_1(x,y) = (x-y)^2$, $d_2(x,y) = \sqrt{|x-y|}$, $d_3(x,y) = |x^2 - y^2|$, $d_4(x,y) = |x - 2y|$, and $d_5 = \frac{|x-y|}{1+|x-y|}$. Which of these functions defines a metric on $\mathbb{R}$?
2. Sketch the unit ball centered at $(0,0)$ with respect to the norms $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$.
3. By modifying one of the above norms, find a norm on $\mathbb{R}^2$ for which this unit ball is a rectangle. Find another norm for which this unit ball is an ellipse.

(G2.2)

Let (X,d) be a metric space, and let $E \subset X$ be a subset. Recall that $p \in X$ is called an accumulation point of E if every neighborhood of p contains infinitely many points of E (or equivalently, one point of $E \setminus \{p\}$).

1. Prove that these two definitions are equivalent.
2. Construct a bounded subset of $\mathbb{R}$ with exactly three accumulation points.

(G2.3)

Let $f : \mathbb{R} \to \mathbb{C}$ be jump continuous on $[-\pi, \pi]$ and 2π-periodic.

1. Show that for any $j, k \in \mathbb{Z}$, $\widehat{e^{-ijx}f}_k = \hat{f}_{k+j}$. (In other words, the kth Fourier coefficient of $e^{-ijx}f(x)$ coincides with the $(k+j)$th Fourier coefficient of $f(x)$.

2. Show that the Fourier coefficient $\widehat{f^{(j)}}_k$ of the jth derivative $f^{(j)}$ is given by $(ik)^j \hat{f}_k$.

3. Let $L > 0$ be a real number. A function $g : \mathbb{R} \to \mathbb{C}$ is called L-periodic if $g(x+L) = g(x)$ for all $x \in \mathbb{R}$. To define a Fourier series for g, let $\tilde{g}(x) = g(\frac{L}{2\pi}x)$. Show that $\tilde{g}$ is 2π-periodic, and that $\tilde{g}_k = \frac{1}{L}\int_{-L/2}^{L/2} g(x)e^{-ik\frac{2\pi}{L}x}\,dx$.